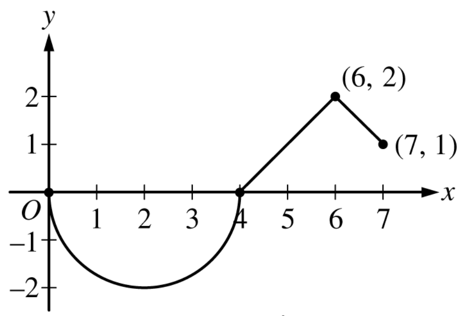


Graph of $f'$

3. Let $f$ be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of $f'$, the derivative of $f$, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

(b) Find the $x$-coordinates of all points of inflection of the graph of $f$ for $0 < x < 7$. Justify your answer.

(c) Let $g$ be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is $g$ decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

(d) For the function $g$ defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

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| $t$ (days) | 0 | 3 | 7 | 10 | 12 |
|---|---|---|---|---|---|
| $r'(t)$ (centimeters per day) | $-6.1$ | $-5.0$ | $-4.4$ | $-3.8$ | $-3.5$ |

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function $r$, where $r(t)$ is measured in centimeters and $t$ is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of $r'$ over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time $t$, $0 \le t \le 3$, for which $r'(t) = -6$ ? Justify your answer.

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume $V$ of a cone with radius $r$ and height $h$ is $V = \frac{1}{3}\pi r^2 h$.)

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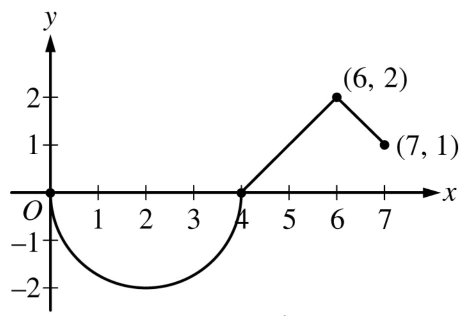
## Part B (AB or BC): Graphing calculator not allowed
## Question 3 — 9 points

**General Scoring Notes**
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of $f'$

Let $f$ be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of $f'$, the derivative of $f$, consists of a semicircle and two line segments, as shown in the figure above.

| Model Solution | Scoring | |
|---|---|---|
| **(a)** Find $f(0)$ and $f(5)$. | | |
| $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ <br> $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$ | Area of either region <br> – OR – $\int_0^4 f'(x)\,dx$ <br> – OR – $\int_4^5 f'(x)\,dx$ | **1 point** |
| | $f(0)$ | **1 point** |
| | $f(5)$ | **1 point** |

**Scoring notes:**

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

**Total for part (a) 3 points**

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**(d)** For the function $g$ defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

| | | |
|---|---|---|
| $g$ is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.<br><br>Therefore, the absolute minimum occurs at $x = 5$, and $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of $g$. | Considers $g'(x) = 0$ | **1 point** |
| | Answer with justification | **1 point** |

**Scoring notes:**

- A justification that uses a local argument, such as "$g'$ changes from negative to positive (or $g$ changes from decreasing to increasing) at $x = 5$" must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
  - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

  $g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5,\ x = 7$

  | $x$ | $g(x)$ |
  |---|---|
  | 0 | $3 + 2\pi$ |
  | 5 | $-\frac{3}{2}$ |
  | 7 | $-\frac{1}{2}$ |

  The absolute minimum value of $g$ on the interval $0 \le x \le 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

**Total for part (d) 2 points**

**Total for question 3 9 points**

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