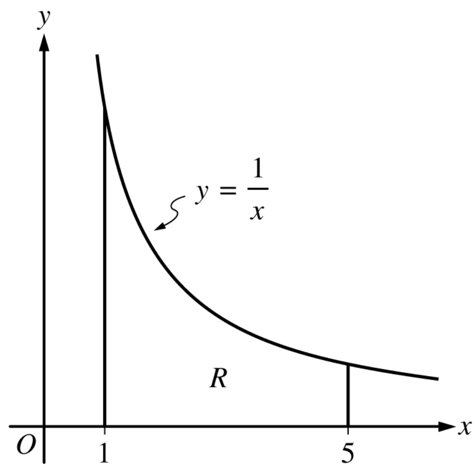

Figure 1

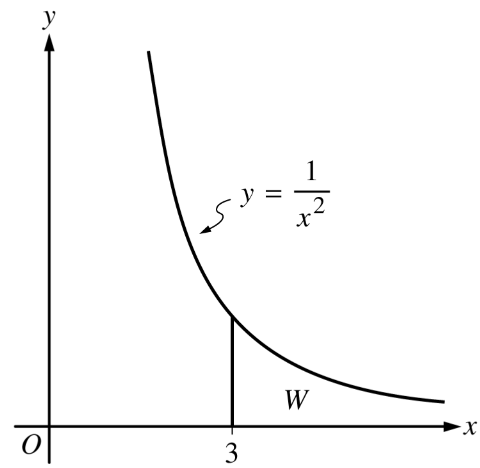

Figure 2

5. Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x-axis that lies to the right of the vertical line $x = 3$.

(a) Find the area of region R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.

(c) Find the volume of the solid generated when the unbounded region W is revolved about the x-axis.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. The function f is defined by the power series $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$ for all real numbers x for which the series converges.

(a) Using the ratio test, find the interval of convergence of the power series for f. Justify your answer.

(b) Show that $\left| f\left(\frac{1}{2}\right) - \frac{1}{2} \right| < \frac{1}{10}$. Justify your answer.

(c) Write the first four nonzero terms and the general term for an infinite series that represents $f'(x)$.

(d) Use the result from part (c) to find the value of $f'\left(\frac{1}{6}\right)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

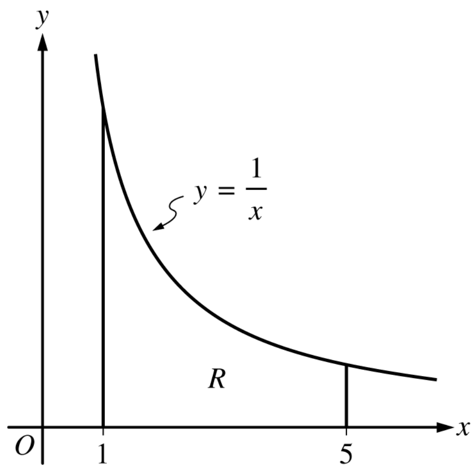

Figure 1

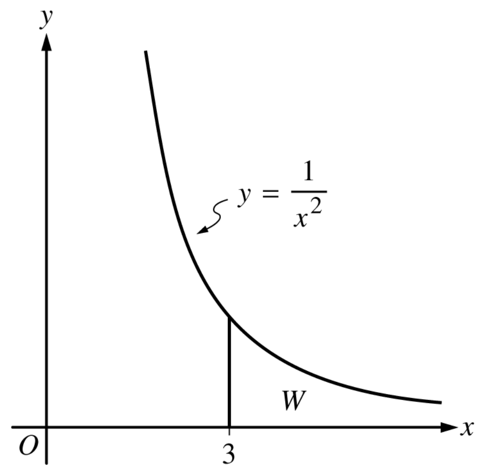

Figure 2

Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x-axis that lies to the right of the vertical line $x = 3$.

	Model Solution	Scoring	
(a)	Find the area of region R.		
	Area $= \int_1^5 \frac{1}{x}\,dx$	Integral	**1 point**
	$= \ln x \Big\vert_1^5$ $= \ln 5 - \ln 1 = \ln 5$	Answer	**1 point**

Scoring notes:

- A definite integral with incorrect bounds does not earn either point.
- An unevaluated indefinite integral does not earn either point.
- An indefinite integral that is evaluated in a later step may earn one or both points. For example, $\int \frac{1}{x}\,dx = \ln 5 - \ln 1$ (or $\ln 5$) does not earn the first point but does earn the second. However, $\int \frac{1}{x}\,dx = \ln x + C \Rightarrow \text{Area} = \ln 5 - \ln 1$ earns both points.

Total for part (a) 2 points

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.

$\text{Volume} = \int_1^5 xe^{x/5}\,dx$	Definite integral	**1 point**
Using integration by parts, $u = x \quad dv = e^{x/5}\,dx$ $du = dx \quad v = 5e^{x/5}$	u and dv	**1 point**
$\int xe^{x/5}\,dx = 5xe^{x/5} - \int 5e^{x/5}\,dx$ $= 5xe^{x/5} - 25e^{x/5} + C$ $= 5e^{x/5}(x-5) + C$	$\int xe^{x/5}\,dx$ $= 5xe^{x/5} - \int 5e^{x/5}\,dx$	**1 point**
$\text{Volume} = 5e^{x/5}(x-5)\Big\vert_1^5$ $= 5e(0) - 5e^{1/5}(-4) = 20e^{1/5}$	Answer	**1 point**

Scoring notes:

- The first point is earned for $c\int_1^5 xe^{x/5}\,dx$, where $c \neq 0$. Errors of $c \neq 1$, for example $c = \pi$, will not earn the fourth point.
- Incorrect integrals that require integration by parts are still eligible for the second and third points. Both of these points will be earned with at least one correct application of integration by parts.
- The second point will be earned with an implied u and dv in the presence of $5xe^{x/5} - \int 5e^{x/5}\,dx$.
- The tabular method may be used to show integration by parts. In this case, the second point is earned by having columns (labeled or unlabeled) that begin with x and $e^{x/5}$. The third point is earned for either $5xe^{x/5} - \int 5e^{x/5}\,dx$ or $5xe^{x/5} - 25e^{x/5}$.
- Limits of integration may be present, omitted, or partially present in the work for the second and third points.
- The fourth point is earned only for the correct answer.

Total for part (b) 4 points

(c) Find the volume of the solid generated when the unbounded region W is revolved about the x-axis.

$\text{Volume} = \pi \int_3^\infty \left(\frac{1}{x^2}\right)^2 dx = \pi \lim_{b\to\infty} \int_3^b \frac{1}{x^4}\, dx$	Improper integral	**1 point**
$= \pi \lim_{b\to\infty} \left(\frac{1}{-3x^3} \Big\vert_3^b \right)$	Antiderivative	**1 point**
$= \pi \lim_{b\to\infty} \left(\frac{1}{-3}\right)\left[\frac{1}{b^3} - \frac{1}{3^3}\right]$ $= \pi\left(\frac{1}{-3}\right)\left(0 - \frac{1}{3^3}\right) = \frac{\pi}{81}$	Answer	**1 point**

Scoring notes:

- The first point is earned for either $c\int_3^\infty \left(\frac{1}{x^2}\right)^2 dx$ or $\lim_{b\to\infty} c\int_3^b \frac{1}{x^4}\, dx,$ where $c \neq 0$. Errors of $c \neq \pi$ will not earn the third point.
- The second point is earned for a correct antiderivative of any integrand of the form $\frac{1}{x^n},$ for any integer $n \geq 2$.
- To earn the answer point, a response must use correct limit notation and cannot include arithmetic with infinity, such as $\frac{1}{\infty^3}$.

Total for part (c) 3 points

Total for question 5 9 points