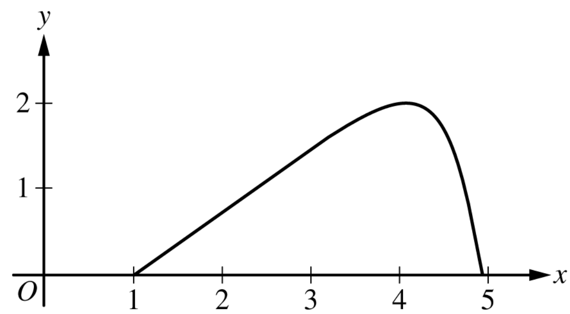


2. For $0 \le t \le \pi$, a particle is moving along the curve shown so that its position at time t is $(x(t), y(t))$, where $x(t)$ is not explicitly given and $y(t) = 2\sin t$. It is known that $\frac{dx}{dt} = e^{\cos t}$. At time $t = 0$, the particle is at position $(1, 0)$.

(a) Find the acceleration vector of the particle at time $t = 1$. Show the setup for your calculations.

(b) For $0 \le t \le \pi$, find the first time t at which the speed of the particle is 1.5. Show the work that leads to your answer.

(c) Find the slope of the line tangent to the path of the particle at time $t = 1$. Find the x-coordinate of the position of the particle at time $t = 1$. Show the work that leads to your answers.

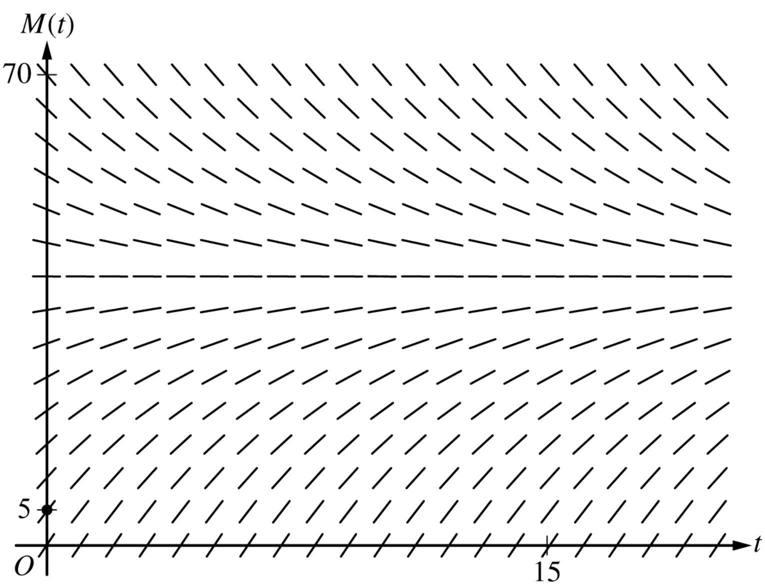
(d) Find the total distance traveled by the particle over the time interval $0 \le t \le \pi$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30), 0.1(30), 0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90-60) + f(120)(120-90) + f(135)(135-120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only 8.25 does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9,$ or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points

(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

f is differentiable. $\Rightarrow$ f is continuous on $[60, 120]$. $\dfrac{f(120) - f(60)}{120 - 60} = \dfrac{0.1 - 0.1}{60} = 0$ By the Mean Value Theorem, there must exist a c, for $60 < c < 120$, such that $f'(c) = 0$.	$f(120) - f(60) = 0$ **1 point** Answer with justification **1 point**

Scoring notes:

- To earn the first point a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must:
  - o have earned the first point,
  - o state that f is continuous because f is differentiable (or equivalent), and
  - o answer "yes" in some way.
- A response may reference either the Mean Value Theorem or Rolle's Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$. Show the setup for your calculations.

$\dfrac{1}{150 - 0}\displaystyle\int_0^{150} g(t)\, dt$	Average value formula	**1 point**
$= 0.0959967$	Answer	**1 point**

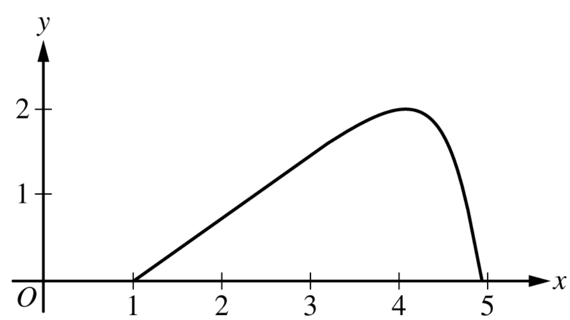
Part A (BC): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



For $0 \le t \le \pi$, a particle is moving along the curve shown so that its position at time t is $(x(t), y(t))$, where $x(t)$ is not explicitly given and $y(t) = 2\sin t$. It is known that $\frac{dx}{dt} = e^{\cos t}$. At time $t = 0$, the particle is at position $(1, 0)$.

	Model Solution	Scoring	
(a)	Find the acceleration vector of the particle at time $t = 1$. Show the setup for your calculations.		
	$x''(1) = \frac{d}{dt}\left(e^{\cos t}\right)\Big\rvert_{t=1} = -1.444407$	$x''(1)$ with setup	**1 point**
	$y(t) = 2\sin t \Rightarrow y'(t) = 2\cos t$ $y''(1) = \frac{d}{dt}(2\cos t)\Big\rvert_{t=1} = -1.682942$	$y''(1)$ with setup	**1 point**
	The acceleration vector at time $t = 1$ is $a(1) = \langle -1.444, -1.683 \text{ (or } -1.682)\rangle$.		

Scoring notes:

- The exact answer is $\langle x''(1), y''(1)\rangle = \left\langle -e^{\cos 1}\sin 1, -2\sin 1\right\rangle$.
- $\left\langle -e^{\cos t}\sin t, -2\sin t\right\rangle$ together with an incorrect or missing evaluation at $t = 1$ earns 1 of the 2 points.

- A response of $\left\langle -e^{\cos t}\sin t, -2\sin t\right\rangle = \left\langle -e^{\cos 1}\sin 1, -2\sin 1\right\rangle$ or equivalent earns only 1 of the 2 points because it equates an expression to a numerical value.
- An unsupported correct acceleration vector earns 1 of the 2 points.
- The acceleration vector may be presented with other symbols, for example $(\, , \,)$ or $[\, , \,]$.
- The components may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x''(1) = -0.000828$ or -0.047433 and $y''(1) = -0.000609$ or -0.034905. A response that presents one of these values with correct setups earns 1 of the 2 points.

Total for part (a) 2 points

(b) For $0 \le t \le \pi$, find the first time t at which the speed of the particle is 1.5. Show the work that leads to your answer.

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}$ $0 \le t \le \pi$ and $\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} = 1.5$	$\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}$ $= 1.5$	**1 point**
$\Rightarrow t = 1.254472,\ t = 2.358077$ The first time at which the speed of the particle is 1.5 is $t = 1.254$.	Answer	**1 point**

Scoring notes:

- A response with an implied equation is eligible for both points. For example, a response of "Speed $= \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}$ and is first equal to 1.5 at $t = 1.254$" earns both points.
- $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.5$ earns the first point. Speed $= 1.5$ by itself does not earn the first point. Both of these responses are eligible to earn the second point.
- A response need not consider the value $t = 2.358077$.
- A response of $t = 1.254$ alone does not earn either point.
- A response with a parenthesis error(s) in either $\left(e^{\cos t}\right)^2$ or $(2\cos t)^2$ does not earn the first point but does earn the second point for the correct answer. Note: $\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$ is not considered a parenthesis error.

- Degree mode: In degree mode, $\sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} = 1.5$ has no solution for $0 \le t \le \pi$.

  A response that finds no time t at which the speed of the particle is 1.5 cannot be assumed to be working in degree mode.

Total for part (b) 2 points

(c) Find the slope of the line tangent to the path of the particle at time $t = 1$. Find the x-coordinate of the position of the particle at time $t = 1$. Show the work that leads to your answers.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{e^{\cos t}}$ $\left.\frac{dy}{dx}\right\|_{t=1} = \frac{2\cos 1}{e^{\cos 1}} = 0.629530$ The slope of the line tangent to the curve at $t = 1$ is 0.630 (or 0.629).	Slope with supporting work	**1 point**
$x(1) = x(0) + \int_0^1 \frac{dx}{dt}\,dt = 1 + \int_0^1 e^{\cos t}\,dt = 3.341575$ The x-coordinate of the position at $t = 1$ is 3.342 (or 3.341).	$\int_0^1 e^{\cos t}\,dt$	**1 point**
	$x(1)$	**1 point**

Scoring notes:

- To earn the first point, the response must communicate $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; for example:
  - $\frac{dy}{dx} = \frac{2\cos 1}{e^{\cos 1}}$
  - $\frac{dy/dt}{dx/dt} = 0.63$
  - $x'(1) = 1.716526,\ \ y'(1) = 1.080605,\ \ \text{slope} = 0.63$
  - $\frac{dy}{dt} = 2\cos t,\ \ \text{slope} = 0.63$
- A response may import an incorrect expression for $y'(t)$ or value of $y'(1)$ from part (a), provided it was declared in part (a).
- The second point is earned for a response that presents the definite integral $\int_0^1 e^{\cos t}\,dt$ or $\int_0^1 \frac{dx}{dt}\,dt$ with or without the initial condition.

- For the second point, if the differential is missing:
  - $\int_0^1 e^{\cos t}$ earns the second point and is eligible for the third point.
  - $x(1) = \int_0^1 e^{\cos t}$ earns the second point but is not eligible for the third point.
  - $x(1) = 1 + \int_0^1 e^{\cos t}$ earns the second point and is eligible for the third point.
  - $x(1) = \int_0^1 e^{\cos t} + 1$ does not earn the second point but earns the third point for the correct answer.
- The third point is not earned for a response that presents an incorrect statement, such as $x(1) = \int_0^1 e^{\cos t}\, dt = 1 + 2.342.$
- Degree mode: In degree mode, $\frac{dy}{dx} = 0.735759$ or 0.012841 and $1 + \int_0^1 e^{\cos t}\, dt = 3.718144.$

Total for part (c) 3 points

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le \pi$. Show the setup for your calculations.

$\int_0^{\pi} \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2}\, dt$	Integral	**1 point**
$= 6.034611$ The total distance traveled by the particle over $0 \le t \le \pi$ is 6.035 (or 6.034).	Answer	**1 point**

Scoring notes:

- The first point is earned for presenting the correct integrand in a definite integral.
- Parentheses errors were assessed in part (b) and, therefore, will not affect the scoring in part (d).
- If the integrand is an incorrect speed function imported from part (b), the response earns the first point and does not earn the second point.
- An unsupported answer of 6.035 (or 6.034) does not earn either point.
- Degree mode: In degree mode, the total distance is 10.596835 or 8.536161.

Total for part (d) 2 points

Total for question 2 9 points

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals.	Intervals	**1 point**
	Reason	**1 point**

Scoring notes:

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f' or the slopes of f'.
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b) 2 points

(c) Find the value of $\lim_{x\to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x\to 2} f(x) = f(2) = 1$. $\lim_{x\to 2}(6f(x) - 3x) = 6\cdot 1 - 3\cdot 2 = 0$ $\lim_{x\to 2}\left(x^2 - 5x + 6\right) = 0$	Limits of numerator and denominator	**1 point**
Because $\lim_{x\to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\frac{0}{0}$, L'Hospital's Rule can be applied. Using L'Hospital's Rule, $\lim_{x\to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x\to 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6\cdot 0 - 3}{2\cdot 2 - 5} = 3.$	Uses L'Hospital's Rule	**1 point**
	Answer	**1 point**

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\frac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c) 3 points