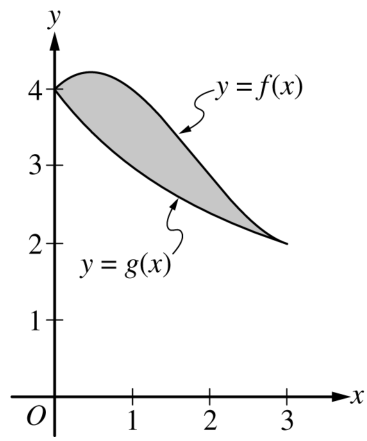


5. The graphs of the functions f and g are shown in the figure for $0 \le x \le 3$. It is known that $g(x) = \dfrac{12}{3+x}$ for $x \ge 0$. The twice-differentiable function f, which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x)\,dx = 10$.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Evaluate the improper integral $\int_0^\infty (g(x))^2\,dx$, or show that the integral diverges.

(c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x)\,dx$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. The function f has derivatives of all orders for all real numbers. It is known that $f(0) = 2$, $f'(0) = 3$, $f''(x) = -f\left(x^2\right)$, and $f'''(x) = -2x \cdot f'\left(x^2\right)$.

(a) Find $f^{(4)}(x)$, the fourth derivative of f with respect to x. Write the fourth-degree Taylor polynomial for f about $x = 0$. Show the work that leads to your answer.

(b) The fourth-degree Taylor polynomial for f about $x = 0$ is used to approximate $f(0.1)$. Given that $\left|f^{(5)}(x)\right| \le 15$ for $0 \le x \le 0.5$, use the Lagrange error bound to show that this approximation is within $\dfrac{1}{10^5}$ of the exact value of $f(0.1)$.

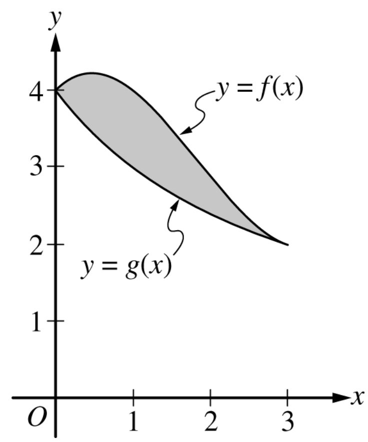
(c) Let g be the function such that $g(0) = 4$ and $g'(x) = e^x f(x)$. Write the second-degree Taylor polynomial for g about $x = 0$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The graphs of the functions f and g are shown in the figure for $0 \le x \le 3$. It is known that $g(x) = \frac{12}{3+x}$ for $x \ge 0$. The twice-differentiable function f, which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x)\,dx = 10$.

Model Solution	Scoring	
(a) Find the area of the shaded region enclosed by the graphs of f and g.		
Area $= \int_0^3 (f(x) - g(x))\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$	Integrand	**1 point**
$= 10 - \int_0^3 \frac{12}{3+x}\,dx = 10 - 12\big[\ln\lvert 3+x\rvert\big]_0^3$	Antiderivative of $g(x)$	**1 point**
$= 10 - 12(\ln 6 - \ln 3) = 10 - 12(\ln 2)$	Answer	**1 point**

Scoring notes:

- The first point is earned for any of the integrands $f(x) - g(x)$, $g(x) - f(x)$, $\lvert f(x) - g(x)\rvert$, or $\lvert g(x) - f(x)\rvert$ in any definite integral. If the limits are incorrect, the response does not earn the third point.

- The first point is earned with an implied integrand for f and explicit integrand for g, such as $10 - \int_0^3 g(x)\,dx$.
- The second point is earned for finding $a\int \frac{dx}{3+x} = a \cdot \ln|3+x|$ or $a \cdot \ln(3+x)$.
- A response is eligible for the third point only if it has earned the first 2 points. The third point is earned only for the correct answer. The answer does not need to be simplified; however, if simplification is attempted, it must be correct.
- A response is not eligible for the third point with incorrect limits of integration for u-substitution, for example, $\int_0^3 \frac{12}{3+x}\,dx = \int_0^3 \frac{12}{u}\,du = 12\left[\ln(x+3)\right]_0^3$.
- A response with incorrect communication, such as "Area $= \int_0^3 (g(x) - f(x))\,dx = 10 - 12(\ln 2)$," does not earn the third point. However, a response of "$\int_0^3 (g(x) - f(x))\,dx = 12(\ln 2) - 10$, so the area is $10 - 12(\ln 2)$" earns all 3 points.

Total for part (a) 3 points

(b) Evaluate the improper integral $\int_0^\infty (g(x))^2\,dx$, or show that the integral diverges.

$\int_0^\infty (g(x))^2\,dx = \lim_{b\to\infty} \int_0^b \frac{144}{(3+x)^2}\,dx$	Limit notation	**1 point**
$= \lim_{b\to\infty} \left(-\frac{144}{(3+x)} \Big\|_0^b \right)$	Antiderivative	**1 point**
$= \lim_{b\to\infty} \left(-\frac{144}{3+b} + \frac{144}{3} \right) = 48$	Answer	**1 point**

Scoring notes:

- To earn the first point a response must correctly use limit notation throughout the problem and not include arithmetic with infinity, for example, $\left[-\frac{144}{3+x}\right]_0^\infty$ or $-\frac{144}{3+\infty} + 48$.
- The second point can be earned by finding an antiderivative of the form $-\frac{a}{(3+x)}$ for $a > 0$, from an indefinite or improper integral, with or without correct limit notation. If $a \neq 144$, the response does not earn the third point.
- The third point is earned only for an answer of 48 (or equivalent).
- A response is not eligible for the third point with incorrect limits of integration for u-substitution, for example, $\lim_{b\to\infty} \int_0^b \frac{144}{u^2}\,du = \lim_{b\to\infty}\left[-\frac{144}{3+x}\right]_0^b$.

Total for part (b) 3 points

(c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x)\,dx$.

Using integration by parts, $u = x \quad dv = f'(x)\,dx$ $du = dx \quad v = f(x)$	u and dv	**1 point**
$\int h(x)\,dx = \int x \cdot f'(x)\,dx = x \cdot f(x) - \int f(x)\,dx$	$\int h(x)\,dx$ $= x \cdot f(x) - \int f(x)\,dx$	**1 point**
$\int_0^3 h(x)\,dx = \int_0^3 x \cdot f'(x)\,dx = x \cdot f(x)\Big\vert_0^3 - \int_0^3 f(x)\,dx$ $= (3 \cdot f(3) - 0 \cdot f(0)) - 10 = 3 \cdot 2 - 0 - 10 = -4$	Answer	**1 point**

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $x \cdot f(x) - \int f(x)\,dx$ or $x \cdot f(x)\Big\vert_0^3 - 10$.
- Limits of integration may be present, omitted, or partially present in the work for the first and second points.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by having columns (labeled or unlabeled) that begin with x and $f'(x)$. The second point is earned for $x \cdot f(x) - \int f(x)\,dx$.
- The third point is earned only for the correct answer and can only be earned if the first 2 points were earned.

Total for part (c) 3 points

Total for question 5 9 points