x	0	π	2π
$f'(x)$	5	6	0

5. The function f is twice differentiable for all x with $f(0) = 0$. Values of f', the derivative of f, are given in the table for selected values of x.

   (a) For $x \geq 0$, the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$. Find the value of $h'(\pi)$. Show the work that leads to your answer.

   (b) What information does $\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx$ provide about the graph of f ?

   (c) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(2\pi)$. Show the computations that lead to your answer.

   (d) Find $\int (t + 5)\cos\left(\frac{t}{4}\right) dt$. Show the work that leads to your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

6. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2 6^n}$ and converges to $f(x)$ for all x in the interval of convergence. It can be shown that the Maclaurin series for f has a radius of convergence of 6.

(a) Determine whether the Maclaurin series for f converges or diverges at $x = 6$. Give a reason for your answer.

(b) It can be shown that $f(-3) = \sum_{n=1}^{\infty} \frac{(n+1)(-3)^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{n+1}{n^2}\left(-\frac{1}{2}\right)^n$ and that the first three terms of this series sum to $S_3 = -\frac{125}{144}$. Show that $\left| f(-3) - S_3 \right| < \frac{1}{50}$.

(c) Find the general term of the Maclaurin series for f', the derivative of f. Find the radius of convergence of the Maclaurin series for f'.

(d) Let $g(x) = \sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{n^2 3^n}$. Use the ratio test to determine the radius of convergence of the Maclaurin series for g.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	π	2π
$f'(x)$	5	6	0

The function f is twice differentiable for all x with $f(0) = 0$. Values of f', the derivative of f, are given in the table for selected values of x.

Model Solution	Scoring	
(a) For $x \geq 0$, the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$. Find the value of $h'(\pi)$. Show the work that leads to your answer.		
$h'(x) = \sqrt{1 + (f'(x))^2}$	Fundamental Theorem of Calculus	**1 point**
$h'(\pi) = \sqrt{1 + (f'(\pi))^2} = \sqrt{1 + 6^2} = \sqrt{37}$	Answer	**1 point**

Scoring notes:

- A response of $\sqrt{1 + (f'(\pi))^2}$ earns the first point.
- A response of $\sqrt{1 + 6^2}$ alone earns both points.
- A response such as $h'(x) = \sqrt{1 + (f'(x))^2} = \sqrt{37}$, that equates a variable expression to a numeric value, earns at most 1 of the 2 points.
- A response that equates $h'(x)$ or $h'(\pi)$ to a derivative of a constant, such as $h'(x) = \frac{d}{dx}\int_0^\pi \sqrt{1 + (f'(t))^2}\, dt$, earns at most 1 of the 2 points.

Total for part (a) 2 points

(b) What information does $\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx$ provide about the graph of f ?

$\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx$ is the arc length of the graph of f on $[0, \pi]$.	Arc length of f	**1 point**
	Interval $[0, \pi]$	**1 point**

Scoring notes:

- A response of "arc length" or "length" earns the first point. Such a response does not need to reference f. However, if the response references a different function, the response does not earn the first point and is eligible to earn the second point.
- A response referring to distance explicitly connected to the graph or f (or equivalent) earns the first point. For example, a response of "distance along the curve" or "distance traveled by a particle moving along f " earns the first point and is eligible to earn the second point.
- A response referring to distance that is not explicitly connected to the graph of f does not earn the first point but is eligible to earn the second point. For example, a response of "distance" or "distance traveled" does not earn the first point but is eligible to earn the second point.
- To earn the second point a response must connect the interval $[0, \pi]$ to arc length, length, or distance.

Total for part (b) 2 points

(c) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(2\pi)$. Show the computations that lead to your answer.

$f(\pi) \approx f(0) + \pi f'(0) = 0 + 5\pi = 5\pi$ $f(2\pi) \approx f(\pi) + \pi f'(\pi)$	Euler's method	**1 point**
$\approx 5\pi + 6\pi = 11\pi$	Answer	**1 point**

Scoring notes:

- To earn the first point a response must demonstrate two Euler's steps, with use of the correct expression for $\frac{dy}{dx}$, and at most one error. If there is an error, the second point is not earned.
- In order to earn the first point, a response that presents a single error in computing the approximation of $f(\pi)$ must import the incorrect value in computing the approximation of $f(2\pi)$.
- The two Euler's steps may be explicit expressions or may be presented in a table. For example:

x	y	$\frac{dy}{dx} \cdot \Delta x$ (or $\frac{dy}{dx} \cdot \pi$)
0	0	5π
π	5π	6π
2π	11π	

  - In the presence of a correct answer, a table does not need to be labeled in order to earn both points. In the presence of no answer or an incorrect answer, such a table must be correctly labeled in order to earn the first point.
- Both points are earned for $5\pi + 6\pi$.
- The response may report the final answer as $(2\pi, 11\pi)$.

Total for part (c) 2 points

(d) Find $\int (t+5)\cos\left(\frac{t}{4}\right) dt$. Show the work that leads to your answer.

$u = t+5 \qquad dv = \cos\left(\frac{t}{4}\right) dt$ $du = dt \qquad v = 4\sin\left(\frac{t}{4}\right)$	u and dv	**1 point**
$\int (t+5)\cos\left(\frac{t}{4}\right) dt = 4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$	$uv - \int v\, du$	**1 point**
$= 4(t+5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$	Answer	**1 point**

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by columns (labeled or unlabeled) that begin with $t+5$ and $\cos\left(\frac{t}{4}\right)$. The second point is earned for $4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The third point is earned only for an expression mathematically equivalent to $4(t+5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$ (such as $4t\sin\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$) in the presence of correct supporting work.
- To earn the third point a response must have a final answer that includes a constant of integration.
- Alternate solution:

$$\int (t+5)\cos\left(\frac{t}{4}\right) dt = \int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$u = t \qquad dv = \cos\left(\frac{t}{4}\right) dt$$

$$du = dt \qquad v = 4\sin\left(\frac{t}{4}\right)$$

$$\int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt = 4t\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$= 4t\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + C$$

  - A response can earn the first and second points for correctly applying integration by parts to $\int t\cos\left(\frac{t}{4}\right) dt$. The tabular method may be used to show integration by parts. The third point is earned for the correct answer.

Total for part (d) 3 points

Total for question 5 9 points