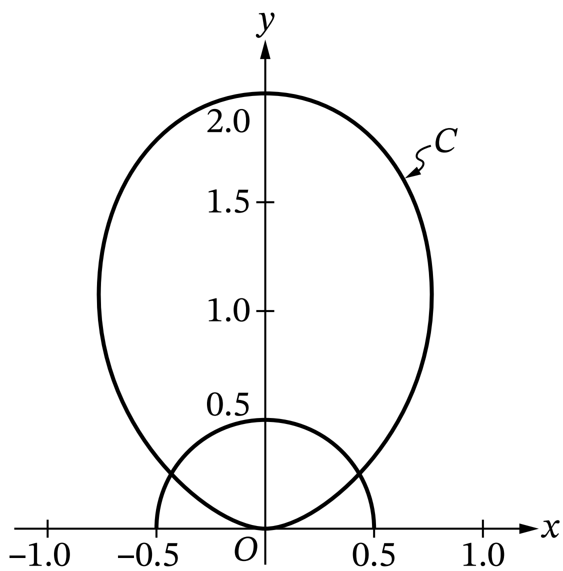
2. Curve C is defined by the polar equation $r(\theta) = 2\sin^2\theta$ for $0 \le \theta \le \pi$. Curve C and the semicircle $r = \frac{1}{2}$ for $0 \le \theta \le \pi$ are shown in the xy-plane.



(Note: Your calculator should be in radian mode.)

A. Find the rate of change of r with respect to θ at the point on curve C where $\theta = 1.3$. Show the setup for your calculations.

B. Find the area of the region that lies inside curve C but outside the graph of the polar equation $r = \frac{1}{2}$. Show the setup for your calculations.

C. It can be shown that $\frac{dx}{d\theta} = 4\sin\theta\cos^2\theta - 2\sin^3\theta$ for curve C. For $0 \le \theta \le \frac{\pi}{2}$, find the value of θ that corresponds to the point on curve C that is farthest from the y-axis. Justify your answer.

D. A particle travels along curve C so that $\frac{d\theta}{dt} = 15$ for all times t. Find the rate at which the particle's distance from the origin changes with respect to time when the particle is at the point where $\theta = 1.3$. Show the setup for your calculations.

END OF PART A

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

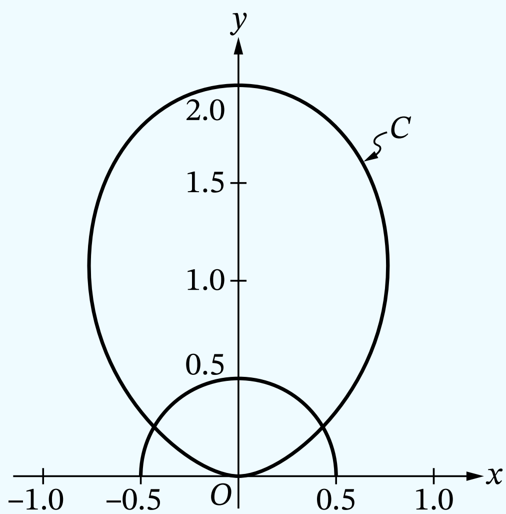
D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

Part A (BC): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Curve C is defined by the polar equation $r(\theta) = 2\sin^2\theta$ for $0 \le \theta \le \pi$. Curve C and the semicircle $r = \frac{1}{2}$ for $0 \le \theta \le \pi$ are shown in the xy-plane.



(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the rate of change of r with respect to θ at the point on curve C where $\theta = 1.3$. Show the setup for your calculations.		
	$\left.\frac{dr}{d\theta}\right\vert_{\theta=1.3} = 1.031003$ The rate of change of r with respect to θ at the point on curve C where $\theta = 1.3$ is 1.031.	Answer with setup	**Point 1 (P1)**

Scoring Notes for Part A

- An exact answer of $\left.\dfrac{dr}{d\theta}\right|_{\theta=1.3} = 4\sin(1.3)\cos(1.3)$ earns **P1**.
- To earn **P1**, a response must indicate differentiation of r and provide the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
  - Examples of responses with correct communication include $\left.\dfrac{dr}{d\theta}\right|_{\theta=1.3} = 1.031$ and $r'(1.3) = 1.031$.
  - Responses with incorrect communication, such as $r'(\theta) = 1.031$, $r' = 1.031$, or $\dfrac{dr}{d\theta} = 1.031$, are sufficient to earn **P1**.

B Find the area of the region that lies inside curve C but outside the graph of the polar equation $r = \dfrac{1}{2}$. Show the setup for your calculations.

For $0 \le \theta \le \pi$, $r(\theta) = \dfrac{1}{2}$ for $\theta = \theta_1 = \dfrac{\pi}{6} = 0.523599$ and $\theta = \theta_2 = \dfrac{5\pi}{6} = 2.617994$.	Integrand including $(r(\theta))^2$	**Point 2 (P2)**
$\dfrac{1}{2}\displaystyle\int_{\theta_1}^{\theta_2}\left((r(\theta))^2 - \left(\dfrac{1}{2}\right)^2\right)d\theta$	Integrand	**Point 3 (P3)**
$= 2.066769$ The area is 2.067 (or 2.066).	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P2** is earned for a definite integral including $(r(\theta))^2$, such as $\displaystyle\int_a^b\left((r(\theta))^2 - \left(\dfrac{1}{2}\right)^2\right)d\theta$ or $\int_a^b (r(\theta))^2\,d\theta$, with or without the differential $d\theta$.
- **P3** is earned for a definite integral (or integrals) with a correct integrand, such as $\displaystyle\int_a^b\left((r(\theta))^2 - \left(\dfrac{1}{2}\right)^2\right)d\theta$ or $\displaystyle\int_a^b (r(\theta))^2\,d\theta - \int_c^d \left(\dfrac{1}{2}\right)^2 d\theta$, with or without the differential $d\theta$.
- The limits $\theta_1 = \dfrac{\pi}{6} = 0.523599$ and $\theta_2 = \dfrac{5\pi}{6} = 2.617994$ and the factor $\dfrac{1}{2}$ are assessed in **P4**, not in **P2** or **P3**.
- **P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_{\pi/6}^{5\pi/6}\left((r(\theta))^2-\left(\frac{1}{2}\right)^2\right)d\theta = 4.133538$ so the area is 2.067.

    Note: This response earns **P2** and **P3** for the integral. It also earns **P4** for the correct answer.
  - $\int_{\pi/6}^{5\pi/6}\left((r(\theta))^2-\left(\frac{1}{2}\right)^2\right)d\theta = 2.067$

    Note: This response earns **P2** and **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- **Special case:** An indefinite integral with a correct integrand does not earn **P2**, earns **P3**, and is eligible to earn **P4** with a correct answer.
- A response of $\int_{\pi/6}^{\pi/2}\left((r(\theta))^2-\left(\frac{1}{2}\right)^2\right)d\theta = 2.067$, using the symmetry of the region, earns **P2**, **P3**, and **P4**.

C It can be shown that $\frac{dx}{d\theta} = 4\sin\theta\cos^2\theta - 2\sin^3\theta$ for curve C. For $0 \le \theta \le \frac{\pi}{2}$, find the value of θ that corresponds to the point on curve C that is farthest from the y-axis. Justify your answer.

For $0 \le \theta \le \frac{\pi}{2}$, the curve C is in the first quadrant. Thus, a point on the curve will be farthest away from the y-axis when the x-coordinate attains its maximum value. This will either occur when $\frac{dx}{d\theta} = 0$ or at an endpoint of the interval $0 \le \theta \le \frac{\pi}{2}$. $\frac{dx}{d\theta} = 0$	Considers $\frac{dx}{d\theta} = 0$	**Point 5 (P5)**
$\Rightarrow\ \theta = 0.955317$ θ | $x(\theta) = r(\theta)\cos\theta$ 0 | 0 0.955317 | 0.769800 $\frac{\pi}{2}$ | 0	Justification	**Point 6 (P6)**
Therefore, the value of θ for which the point on the curve is farthest from the y-axis is 0.955.	Answer with supporting work	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for considering $\frac{dx}{d\theta} = 0$. **P5** is not earned by just presenting $\theta = 0.955317$.

  A response that discusses the sign of $\frac{dx}{d\theta}$ changing or uses the phrase "critical points of $x(\theta)$" also earns **P5**.
- The value $\theta = 0.955317$ might be presented as $\arccos\left(\frac{1}{\sqrt{3}}\right)$, $\arcsin\left(\sqrt{\frac{2}{3}}\right)$, or $\arctan\left(\sqrt{2}\right)$.
- To earn **P6** using a candidates test, a response must make a global argument by correctly evaluating $x(\theta)$ at $\theta = 0$, $\theta = 0.955317$, and $\theta = \frac{\pi}{2}$. The evaluations must be correct to the first digit after the decimal, rounded or truncated.
- Alternate justifications:
  - $\frac{dx}{d\theta} > 0$ for $0 < \theta < 0.955$, and $\frac{dx}{d\theta} < 0$ for $0.955 < \theta < \frac{\pi}{2}$. Therefore, $\theta = 0.955$ is the location of the absolute maximum for $x(\theta)$ on the interval $0 \le \theta \le \frac{\pi}{2}$.
  - Because $\frac{dx}{d\theta}$ changes sign from positive to negative at $\theta = 0.955$ (this might be presented as "$\frac{dx}{d\theta} > 0$ for $\theta < 0.955$, and $\frac{dx}{d\theta} < 0$ for $\theta > 0.955$"), it is the location of a relative maximum for $x(\theta)$. And because $\theta = 0.955$ is the only critical point of $x(\theta)$ in the interval $0 \le \theta \le \frac{\pi}{2}$, it is the location of the absolute maximum for $x(\theta)$ on the interval.
  - Because $\left.\frac{dx}{d\theta}\right|_{\theta=0.955} = 0$ and $\left.\frac{d^2x}{d\theta^2}\right|_{\theta=0.955} < 0$, $\theta = 0.955$ is the location of a relative maximum for $x(\theta)$. And because $\theta = 0.955$ is the only critical point of $x(\theta)$ in the interval $0 \le \theta \le \frac{\pi}{2}$, it is the location of the absolute maximum for $x(\theta)$ on the interval.
- A response that presents only a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P6** but is eligible for **P7** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

D	A particle travels along curve C so that $\frac{d\theta}{dt} = 15$ for all times t. Find the rate at which the particle's distance from the origin changes with respect to time when the particle is at the point where $\theta = 1.3$. Show the setup for your calculations.		
	$\left.\frac{dr}{dt}\right\rvert_{\theta=1.3} = \left.\left(\frac{dr}{d\theta}\cdot\frac{d\theta}{dt}\right)\right\rvert_{\theta=1.3} = 1.031003\cdot 15 = 15.465041$	$\frac{dr}{d\theta}\cdot\frac{d\theta}{dt}$	**Point 8 (P8)**
	The particle's distance from the origin changes at a rate of 15.465.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P8**, $\frac{dr}{d\theta}\cdot\frac{d\theta}{dt}$ can be presented either symbolically or numerically.
- **P8** might be earned in one or more steps.
- A response of $1.031\cdot 15$ or $[\text{answer from part A}]\cdot 15$ earns both **P8** and **P9**.
- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.