5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (3 - x)y^2$ with initial condition $f(1) = -1$.

A. Find $f''(1)$, the value of $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. Show the work that leads to your answer.

B. Write the second-degree Taylor polynomial for f about $x = 1$.

C. The second-degree Taylor polynomial for f about $x = 1$ is used to approximate $f(1.1)$. Given that $|f'''(x)| \le 60$ for all x in the interval $1 \le x \le 1.1$, use the Lagrange error bound to show that this approximation differs from $f(1.1)$ by at most 0.01.

D. Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the work that leads to your answer.

6. The Taylor series for a function f about $x = 4$ is given by
$\sum_{n=1}^{\infty} \frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2 \cdot 3} + \frac{(x-4)^3}{3 \cdot 3^2} + \frac{(x-4)^4}{4 \cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$ and converges to $f(x)$ on its interval of convergence.

A. Using the ratio test, find the interval of convergence of the Taylor series for f about $x = 4$. Justify your answer.

B. Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 4$.

C. The Taylor series for f' described in part B is a geometric series. For all x in the interval of convergence of the Taylor series for f', show that $f'(x) = \frac{x-4}{7-x}$.

D. It is known that the radius of convergence of the Taylor series for f about $x = 4$ is the same as the radius of convergence of the Taylor series for f' about $x = 4$. Does the Taylor series for f' described in part B converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.

STOP

END OF EXAM

Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (3 - x)y^2$ with initial condition $f(1) = -1$.

	Model Solution	Scoring	
A	Find $f''(1)$, the value of $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. Show the work that leads to your answer.		
	$\frac{d^2y}{dx^2} = -y^2 + (3 - x)2y\frac{dy}{dx}$	Product rule	**Point 1 (P1)**
		Chain rule	**Point 2 (P2)**
	$f'(1) = \left.\frac{dy}{dx}\right\rvert_{(x,y)=(1,\,-1)} = (3-1)(-1)^2 = 2$ $f''(1) = \left.\frac{d^2y}{dx^2}\right\rvert_{(x,y)=(1,\,-1)} = -(-1)^2 + (3-1)(2)(-1)(2) = -9$	$f''(1)$	**Point 3 (P3)**