6. The Taylor series for a function f about $x = 4$ is given by $\sum_{n=1}^{\infty} \frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2 \cdot 3} + \frac{(x-4)^3}{3 \cdot 3^2} + \frac{(x-4)^4}{4 \cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$ and converges to $f(x)$ on its interval of convergence.

A. Using the ratio test, find the interval of convergence of the Taylor series for f about $x = 4$. Justify your answer.

B. Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 4$.

C. The Taylor series for f' described in part B is a geometric series. For all x in the interval of convergence of the Taylor series for f', show that $f'(x) = \frac{x-4}{7-x}$.

D. It is known that the radius of convergence of the Taylor series for f about $x = 4$ is the same as the radius of convergence of the Taylor series for f' about $x = 4$. Does the Taylor series for f' described in part B converge to $f'(x) = \frac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.

STOP

END OF EXAM