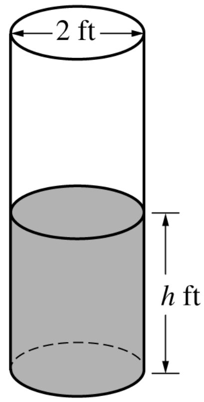

4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

   (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

   (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

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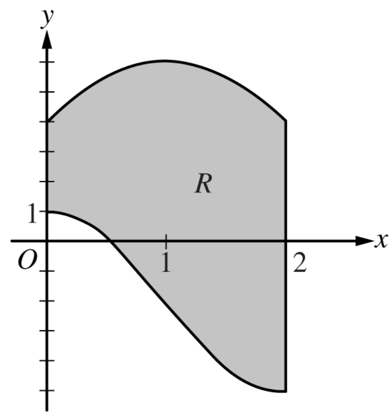

5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

Question 4

(a) $V = \pi r^2 h = \pi (1)^2 h = \pi h$

$$\left.\frac{dV}{dt}\right|_{h=4} = \pi \left.\frac{dh}{dt}\right|_{h=4} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5} \text{ cubic feet per second}$$

$2 : \begin{cases} 1 : \dfrac{dV}{dt} = \pi \dfrac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$

(b) $$\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$$

Because $\dfrac{d^2h}{dt^2} = \dfrac{1}{200} > 0$ for $h > 0,$ the rate of change of the height is increasing when the height of the water is 3 feet.

$3 : \begin{cases} 1 : \dfrac{d}{dh}\left(-\dfrac{1}{10}\sqrt{h}\right) = -\dfrac{1}{20\sqrt{h}} \\ 1 : \dfrac{d^2h}{dt^2} = -\dfrac{1}{20\sqrt{h}} \cdot \dfrac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$

(c) $$\frac{dh}{\sqrt{h}} = -\frac{1}{10}\,dt$$

$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$$

$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \quad\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$

Note: $0/4$ if no separation of variables

Note: max $2/4$ [1-1-0-0] if no constant of integration