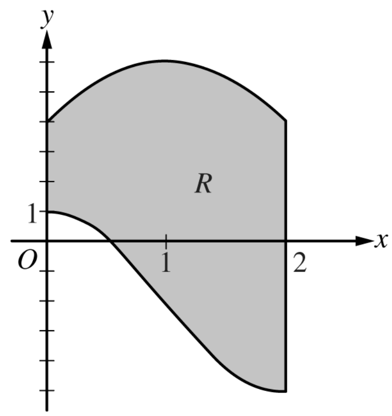


5. Let $R$ be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the $y$-axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of $R$.

(b) Region $R$ is the base of a solid. For the solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 6$.

6. Functions $f$, $g$, and $h$ are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of $g$ at $x = 2$ and the graph of $h$ at $x = 2$.

   (a) Find $h'(2)$.

   (b) Let $a$ be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

   (c) The function $h$ satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

   (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let $k$ be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is $k$ continuous at $x = 2$ ? Justify your answer.

---

**STOP**

**END OF EXAM**

## Question 5

(a) $\int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left( \left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right) \right) dx$

$$= \left[ \left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right) \right]_{x=0}^{x=2}$$

$$= \left( \left(12 - \frac{2}{3}\right) - (-4 + 0) \right) - \left( \left(0 + \frac{2}{3}\right) - (0 + 0) \right)$$

$$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$$

The area of $R$ is $\frac{44}{3}$.

4 :
- 1 : integrand
- 1 : antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$
- 1 : antiderivative of remaining terms
- 1 : answer

(b) $\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$

$$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$$

The volume of the solid is $\ln 5 - \ln 3$.

2 :
- 1 : integral
- 1 : answer

(c) $\pi \int_0^2 \left( (6 - g(x))^2 - (6 - h(x))^2 \right) dx$

3 :
- 1 : limits and constant
- 1 : form of integrand
- 1 : integrand