2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

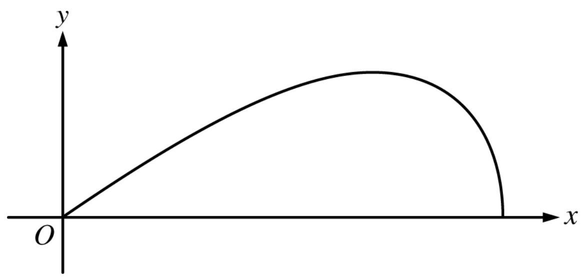
(a) Find the positions of particles P and Q at time $t = 1$.

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\dfrac{dy}{dx} = \dfrac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

	Model Solution	Scoring	
(a)	Find the positions of particles P and Q at time $t = 1$.		
	$x_P(1) = 5 + \int_0^1 v_P(t)\,dt = 5.370660$ At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).	One definite integral	**1 point**
		One position	**1 point**
	$x_Q(1) = 10 + \int_0^1 v_Q(t)\,dt = 8.564355$ At time $t = 1$, the position of particle Q is $x = 8.564$.	The other position	**1 point**

Scoring notes:

- The first point is earned for the explicit presentation of at least one definite integral, either $\int_0^1 v_P(t)\,dt$ or $\int_0^1 v_Q(t)\,dt$.
- The first point must be earned to be eligible for the second and third points.
- The second point is earned for adding the initial condition to at least one of the definite integrals and finding the correct position.
- Writing $\int_0^1 v_P(t) + 5 = 5.370660$ does not earn a position point, because the missing dt makes this statement unclear or false. However, $5 + \int_0^1 v_P(t) = 5.370660$ does earn the position point because it is not ambiguous. Similarly, for the position of Q.
- Read unlabeled answers presented left to right, or top to bottom, as $x_P(1)$ and $x_Q(1)$, respectively.
- Special case 1: A response of $x_P(1) = 5 + \int_0^a v_P(t)\,dt = 5.370660$ AND $x_Q(1) = 10 + \int_0^a v_Q(t)\,dt = 8.564355$ for $a \neq 1$ earns one point.
- Special case 2: A response of $x_P(1) = 5 + \int v_P(t)\,dt = 5.370660$ AND $x_Q(1) = 10 + \int v_Q(t)\,dt = 8.564355$ or the equivalent, never providing the definite integrals, earns one point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x_P(1)$ is 5.007 (or 5.006).

Total for part (a) 3 points

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$v_P(1) = \sin\left(1^{1.5}\right) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	**1 point**
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q. Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	**1 point**

Scoring notes:

- The first point is earned for using the sign of $v_P(1)$ or $v_Q(1)$ to determine the direction of motion for one of the particles. This point cannot be earned without reference to the sign of $v_P(1)$ or $v_Q(1)$.
- It is not necessary to present an explicit value for $v_P(1)$, or $v_Q(1)$, but if a value is presented, it must be correct as far as reported, up to three places after the decimal.
- Read with imported incorrect position values from part (a).
- If one or both position values were not found in part (a), but are found in part (b), the points for part (a) are not earned retroactively.
- To earn the second point the explanation must be based on the signs of $v_P(1)$ and $v_Q(1)$ and the relative positions of particle P and particle Q at $t = 1$. References to other values of time, such as $t = 0$, are not sufficient.
- Degree mode: $v_P(1) = 0.017$. (See degree mode statement in part (a).)

Total for part (b) 2 points

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$a_Q(1) = v'_Q(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	**1 point**
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	**1 point**

Scoring notes:

- To earn the first point the acceleration must be explicitly connected to v'_Q (e.g., $v'_Q(1) = 1.026856$).
- The first point is not earned for an unsupported value of 1.027 (or 1.026). The setup, $v'_Q(1)$, must be shown. Presenting only $a_Q(1) = 1.027$ (or 1.026) without indication that $v'_Q = a_Q$ is not enough to earn the first point.
- A response does not need to present a value for $v_Q(1)$; the sign is sufficient.
- To earn the second point a response must compare the signs of a_Q and v_Q at $t = 1$. Considering only one sign is not sufficient.
- After the first point has been earned, a response declaring only "velocity and acceleration are of opposite signs at $t = 1$ so the particle is slowing down" (or equivalent) earns the second point.
- The second point may be earned without the first, as long as the response does not present an incorrect value or sign for $v_Q(1)$ and concludes the particle is slowing down because velocity and acceleration have opposite signs at $t = 1$.

Total for part (c) 2 points

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

$\int_0^{\pi} \lvert v_P(t) \rvert \, dt = 1.93148$ Over the time interval $0 \le t \le \pi$, the total distance traveled by particle P is 1.931.	Definite integral Answer	**1 point** **1 point**

Scoring notes:

- The first point is earned for $\int_0^{\pi} \lvert v_P(t) \rvert \, dt$.
- The first point can also be earned for a sum (or difference) of definite integrals, such as $\int_0^{2.145029} v_P(t) \, dt - \int_{2.145029}^{\pi} v_P(t) \, dt$, provided the response has indicated $v_P(2.145029) = 0$.
- The second point can only be earned for the correct answer.
- The unsupported value 1.931 earns no points.
- A response reporting the distance traveled by particle Q as $\int_0^{\pi} \lvert v_Q(t) \rvert \, dt = 3.506$ earns the first point and is not eligible for the second point.
- In degree mode, the total distance traveled is 0.122. (See degree mode statement in part (a).) In the degree mode case, the response must present $\int_0^{\pi} \lvert v_P(t) \rvert \, dt$ in order to earn the first point because $\int_0^{\pi} \lvert v_P(t) \rvert \, dt = \int_0^{\pi} v_P(t) \, dt$.

Total for part (d) 2 points

Total for question 2 9 points

(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	**1 point**

Scoring notes:

- To earn the point the interpretation must include "medication in the patient," "approaches 12," and units (milligrams), or their equivalents.

Total for part (b) **1 point**

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	**1 point**
$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln\lvert 12 - y\rvert = \frac{t}{3} + C$	Antiderivatives	**1 point**
$\ln\lvert 12 - y\rvert = -\frac{t}{3} - C \Rightarrow \lvert 12 - y\rvert = e^{-t/3-C}$ $\Rightarrow y = 12 + Ke^{-t/3}$ $0 = 12 + K \Rightarrow K = -12$	Constant of integration and uses initial condition	**1 point**
$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	**1 point**

Scoring notes:

- A response of $\frac{dy}{12 - y} = 3\,dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.
- Absolute value bars are not required in this part.
- A response that correctly separates to $\frac{3\,dy}{12 - y} = dt$ but then incorrectly simplifies to $\frac{dy}{4 - y} = dt$ earns the first point (for the initial correct separation), is eligible for the second point (for $-\ln\lvert 4 - y\rvert = t$, with or without $+C$), but is not eligible for the third or fourth points.
- $+\ln\lvert 12 - y\rvert = \frac{t}{3}$ (with or without $+C$) does not earn the second point and is not eligible for the fourth point; $+\ln\lvert 12 - y\rvert = \frac{t}{3} + C$ is eligible for the third point.
- In all other cases, the points are earned consecutively—the second point cannot be earned without the first, the third without the second, etc.

Total for part (c) **4 points**