6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle P moves along the x-axis such that, for time $t > 0,$ its position is given by $x_P(t) = 6 - 4e^{-t}.$

Particle Q moves along the y-axis such that, for time $t > 0,$ its velocity is given by $v_Q(t) = \frac{1}{t^2}.$ At time $t = 1,$ the position of particle Q is $y_Q(1) = 2.$

	Model Solution	Scoring	
(a)	Find $v_P(t),$ the velocity of particle P at time $t.$		
	$v_P(t) = x_P'(t) = 4e^{-t}$	Answer	**1 point**

Scoring notes:

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

Total for part (a) — 1 point

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

$a_Q(t) = v_Q{}'(t) = \frac{-2}{t^3}$	$a_Q(t)$	**1 point**
For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$.	Considers signs of $a_Q(t)$ and $v_Q(t)$	**1 point**
Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.	Answer with justification	**1 point**

Scoring notes:

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: "$v_Q(t)$ and $a_Q(t)$ have opposite signs" or "$v_Q(t)$ and $a_Q(t)$ have the same sign." This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

Total for part (b) 3 points

(c) Find $y_Q(t)$, the position of particle Q at time t.

$y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 2 - \left(\frac{1}{s} \Big\|_1^t \right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	Answer	**1 point**

Scoring notes:

- A response that presents $\int_1^t \frac{1}{t^2}\,dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2}\,dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

Total for part (c) 3 points

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

For particle P, $\lim\limits_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$. For particle Q, $\lim\limits_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$.	One correct limit	**1 point**
Because $6 > 3$, particle P will eventually be farther from the origin.	Answer with reason	**1 point**

Scoring notes:

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle P and the presented $y_Q(t)$.
- Responses that present statements such as "$6 - 4e^{-t}$ approaches 6" or "Q goes to 3" earn the first point and are eligible for the second point.
- A response that treats ∞ as an input for $x_P(t)$ or $y_Q(t)$, such as "$6 - 4e^{-\infty}$" or "$3 - \frac{1}{\infty}$" is not eligible for the second point.

Total for part (d) 2 points

Total for question 6 9 points