2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

(a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

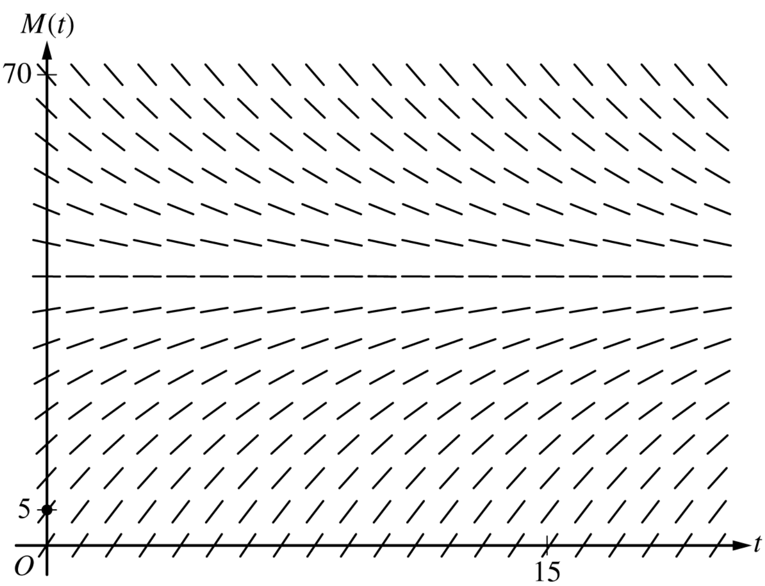
(d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30),\ 0.1(30),\ 0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90-60) + f(120)(120-90) + f(135)(135-120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only 8.25 does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9$, or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points

(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

f is differentiable. $\Rightarrow$ f is continuous on $[60, 120]$.	$f(120) - f(60) = 0$	**1 point**
$\dfrac{f(120) - f(60)}{120 - 60} = \dfrac{0.1 - 0.1}{60} = 0$ By the Mean Value Theorem, there must exist a c, for $60 < c < 120$, such that $f'(c) = 0$.	Answer with justification	**1 point**

Scoring notes:

- To earn the first point a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must:
  - o have earned the first point,
  - o state that f is continuous because f is differentiable (or equivalent), and
  - o answer "yes" in some way.
- A response may reference either the Mean Value Theorem or Rolle's Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$. Show the setup for your calculations.

$\dfrac{1}{150 - 0}\displaystyle\int_0^{150} g(t)\,dt$	Average value formula	**1 point**
$= 0.0959967$	Answer	**1 point**

Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

	Model Solution	Scoring	
(a)	Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.		
	For $0 < t < 90$, $v(t) = 0 \Rightarrow t = 56$.	Considers sign of $v(t)$	**1 point**
	Stephen changes direction when his velocity changes sign. This occurs at $t = 56$ seconds.	Answer with reason	**1 point**

Scoring notes:

- A response that considers $v(t) = 0$ earns the first point.
- A response of "Stephen changes direction when his velocity changes sign" earns the first point for considering the sign of $v(t)$ but must include the answer of $t = 56$ in order to earn the second point.
- A response of $t = 56$ with no supporting work does not earn either point.
- Any presented values of t outside the interval $0 < t < 90$ will not affect scoring.

Total for part (a) — 2 points

	Model Solution	Scoring	
(b)	Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.		
	$v'(60) = a(60) = -0.0360162$	$a(60)$ with setup	**1 point**
	Stephen's acceleration at time $t = 60$ seconds is -0.036 meter per second per second.	Acceleration units	**1 point**

$v(60) = -0.1595124 < 0$ Stephen is speeding up at time $t = 60$ seconds because Stephen's velocity and acceleration are both negative at that time.	Speeding up with reason	**1 point**

Scoring notes:

- The minimum work needed to earn the first point is $v'(60) = -0.036$.
    - $a(60) = -0.0360162$ is not sufficient to earn the first point. The connection $v'(t) = a(t)$ or $v'(60) = a(60)$ must be explicitly shown.
- A response must declare a value for $a(60)$ to be eligible for the second point.
- In order to earn the third point the presented conclusion must be consistent with a negative velocity at time $t = 60$ and the presented value of $a(60)$.
- A response does not need to find the value of $v(60)$; an implied sign is sufficient.
    - The statement "Stephen is speeding up because $a(60)$ and $v(60)$ have the same sign" (or equivalent) earns the third point, provided a negative value is presented for $a(60)$.
- A response that reports an incorrect sign or value of $v(60)$ does not earn the third point. Any presented value of $v(60)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the third point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode).
    - In degree mode, there are two possible values for $v'(60)$. A response that declares $v'(60) = -0.000141$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is slowing down.
    - Similarly, a response that declares $v'(60) = 0.039304$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is speeding up.

Total for part (b) 3 points

(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

$\int_{20}^{80} v(t)\,dt$	Integral	**1 point**
$= 23.383997$ The distance between Stephen's positions at $t = 20$ seconds and $t = 80$ seconds is 23.384 (or 23.383) meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_{20}^{80} v(t)\, dt$ (or the equivalent) with or without the differential.
- The second point is earned only for an answer of 23.384 (or 23.383) regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{20}^{80} v(t)\, dt = 2.407982.$

Total for part (c) 2 points

(d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

$\int_0^{90} \lvert v(t) \rvert\, dt$	Integral	**1 point**
$= 62.164216$ The total distance Stephen swims over the time interval $0 \le t \le 90$ seconds is 62.164 meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_0^{90} \lvert v(t) \rvert\, dt$ or $\int_0^{56} v(t)\, dt - \int_{56}^{90} v(t)\, dt$ (or the equivalent), with or without the differential(s).
- The second point is earned only for an answer of 62.164 regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_0^{90} \lvert v(t) \rvert\, dt = 3.127892.$

Total for part (d) 2 points

Total for question 2 9 points