2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

   (a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

   (b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

   (c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

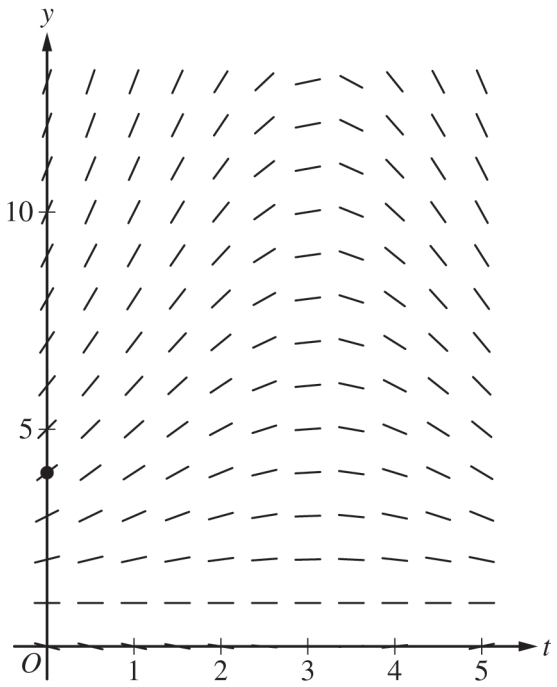
   (d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Scoring notes:

- Read "=" as "≈" for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3\cdot 100 + 4\cdot 85 + 5\cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3\cdot 85 + 4\cdot 69 + 5\cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average temperature" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

	Model Solution	Scoring	
(a)	There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.		
	$v(t) = 0 \Rightarrow t = 1.425610$ Therefore, the particle is at rest (not moving) at $t_R = 1.426$ (or 1.425).	t_R	**1 point**
	For $0 < t < t_R$, $v(t) > 0$. Therefore, the particle is moving to the right on that interval.	Direction with explanation	**1 point**

Scoring notes:

- The first point is earned for considering $v(t) = 0$ and reporting the value $t = 1.426$ (or $t = 1.425$).
- A response that finds no value or an incorrect value of t_R is not eligible to earn the second point.
- A response need not demonstrate an evaluation of $v(t)$ at any value of t, $0 < t < 1.426$. The evaluations do not need to be presented with three correct decimal places, but they must be correct for the number of digits shown for one up to three digits after the decimal.

Total for part (a) 2 points

(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

$a(1.5) = v'(1.5) = -1$ The acceleration of the particle at time $t = 1.5$ is -1 (or -0.999).	Acceleration with setup	**1 point**
$v(1.5) = -0.076856 < 0$ Because $a(1.5)$ and $v(1.5)$ have the same sign, the speed is increasing at time $t = 1.5$.	Answer with explanation	**1 point**

Scoring notes:

- A response must demonstrate the relationship $v' = a$ in order to earn the first point. This relationship could be shown by "$v'(1.5) = -1$ (or -0.999)" or "$v' = a$ and $a(1.5) = -1$ (or -0.999)." A response of just "$a(1.5) = -1$ (or -0.999)" is not sufficient to earn the first point.
- A response must declare a value for $a(1.5)$ to be eligible for the second point.
- The second point can only be earned for a response that is consistent with a negative velocity at time $t = 1.5$ and the presented value of $a(1.5)$.
- Any presented value of $v(1.5)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the second point.
- A response does not need to report a value for $v(1.5)$; an implied sign is sufficient:
  - Any statement equivalent to "The speed of the particle is increasing because $a(1.5)$ and $v(1.5)$ have the same sign" will earn the second point, provided the presented value of $a(1.5)$ is negative.
- A response that presents or references an incorrect value or sign of $v(1.5)$ does not earn the second point.
- Alternate solution for the second point:

  Speed $= |v(t)|$ and its derivative is positive when $t = 1.5$, therefore the speed of the particle is increasing.

Total for part (b) 2 points

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

$x(4) = x(1) + \int_1^4 v(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= -3 + 0.197117 = -2.802883$ The position of the particle at time $t = 4$ is -2.803 (or -2.802).	Answer	**1 point**

Scoring notes:

- The first point is earned for a definite integral with integrand $v(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A response with a linkage error such as $x(4) = \int_1^4 v(t)\,dt = -3 + 0.197$ or $\int_1^4 v(t)\,dt = 0.197 = -2.803$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
  - Unambiguous responses of $x(1) + \int_1^4 v(t)$ and $-3 + \int_1^4 v(t)$ both earn the first 2 points and are eligible for the third point.
  - Ambiguous responses of $\int_1^4 v(t) + x(1)$ and $\int_1^4 v(t) - 3$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, neither of these responses earn the third point.
- The second point is earned for either symbolically or numerically adding $x(1)$ or -3 to a definite integral with a lower limit of 1.
- The third point is earned for an answer of $-3 + 0.197$ or $0.197 - 3$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just -2.803 (or -2.802) with no supporting work does not earn any points.

Total for part (c) 3 points

(d) Find the total distance traveled by the particle over the interval $1 \le t \le 4$. Show the setup for your calculations.

$\int_1^4 \lvert v(t) \rvert \, dt$	Integral	**1 point**
$= 0.9581$ The total distance traveled by the particle over the interval $1 \le t \le 4$ is 0.958.	Answer	**1 point**

Scoring notes:

- The first point is earned for any one of the following:
  - $\int_1^4 \lvert v(t) \rvert \, dt$
  - $\int_1^{1.425} v(t) \, dt - \int_{1.425}^{2.883} v(t) dt + \int_{2.883}^4 v(t) \, dt$
  - $\int_1^{1.426} v(t) \, dt - \int_{1.426}^{2.883} v(t) dt + \int_{2.883}^4 v(t) \, dt$
- Due to variations in numerical integration techniques on some calculators, responses of 0.958, 0.959, or 0.96 earn the second point.

Total for part (d) 2 points

Total for question 2 9 points