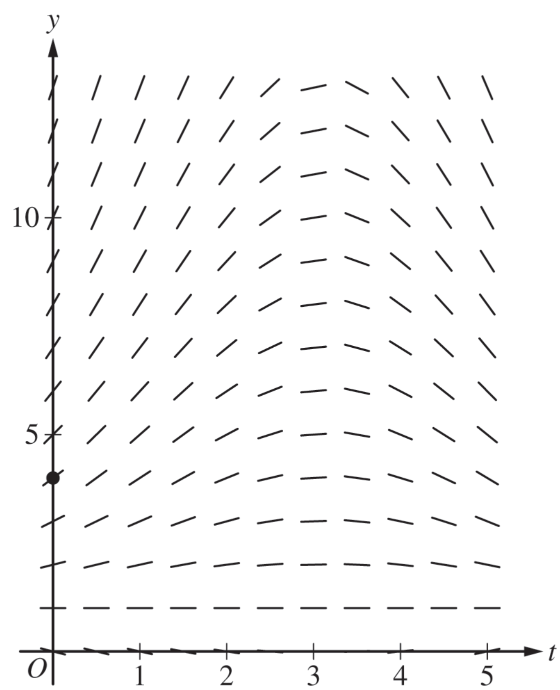
3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

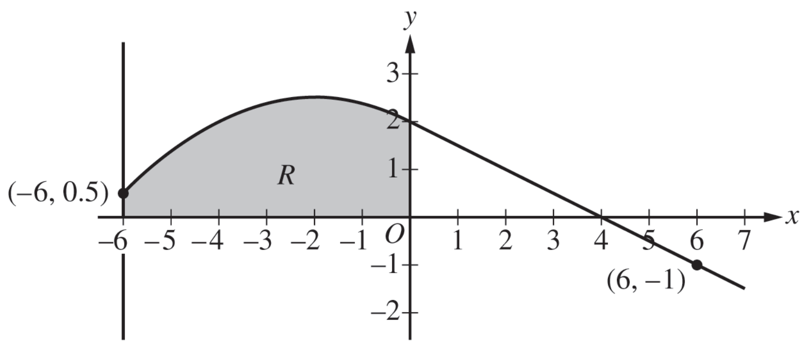
(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\, dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

(c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\, dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

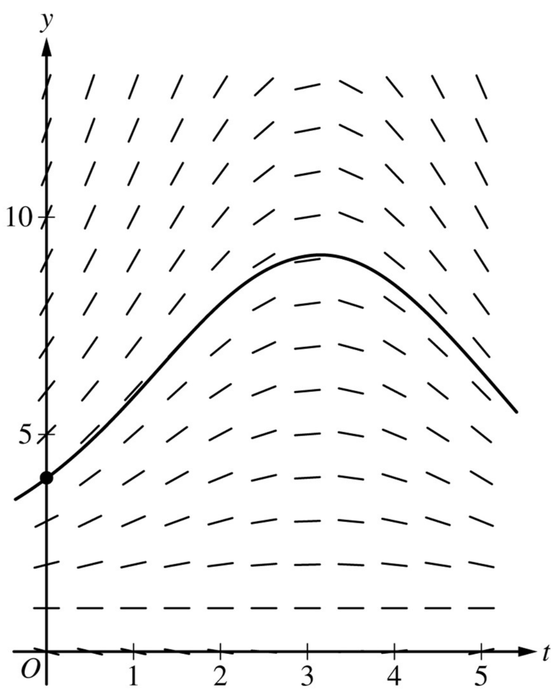
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

	Model Solution	Scoring	
(a)	A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.		
		Solution curve	**1 point**

Scoring notes:

- The solution curve must pass through the point $(0, 4)$, extend to at least $t = 4.5$, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.

Total for part (a) — 1 point

(b) For $0 < t < 5,$ it can be shown that $H(t) > 1.$ Find the value of $t,$ for $0 < t < 5,$ at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Because $H(t) > 1,$ then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0.$	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi,$ $\frac{dH}{dt} > 0$ and for $\pi < t < 5,$ $\frac{dH}{dt} < 0.$ Therefore, $t = \pi$ is the location of a relative maximum value of $H.$	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for considering $\frac{dH}{dt} = 0,$ $\frac{dH}{dt} > 0,$ $\frac{dH}{dt} < 0,$ $\cos\left(\frac{t}{2}\right) = 0,$ $\cos\left(\frac{t}{2}\right) > 0,$ or $\cos\left(\frac{t}{2}\right) < 0.$
- The second point is earned for identifying $t = \pi,$ with or without supporting work. A response may consider $H = 1$ or $t = 1$ as potential critical points without penalty.
- The third point cannot be earned without the first point. The third point is earned only for a correct justification and a correct answer of "relative maximum."
- The justification can be shown by determining the sign of $\frac{dH}{dt}$ (or $\cos\left(\frac{t}{2}\right)$) at a single value in $0 < t < \pi$ and at a single value in $\pi < t < 5.$ It is not necessary to state that $\frac{dH}{dt}$ does not change sign on these intervals.
- The third point can also be earned by using the Second Derivative Test. For example:

  $$\frac{d^2H}{dt^2} = \frac{1}{2}(H-1)\left(-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right) + \cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}\cdot\frac{dH}{dt}$$

  $$\left.\frac{d^2H}{dt^2}\right|_{t=\pi} < 0$$

  Therefore, $t = \pi$ is the location of a relative maximum value of $H.$

Total for part (b) 3 points

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$ $\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	One antiderivative	**1 point**
	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H - 1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response that presents $\int \frac{dH}{H-1} = \ln(H-1)$ without absolute value symbols earns that antiderivative point.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 0 for t and 4 for H.
- A response is eligible for the fifth point only if it has earned the first 4 points.
- A response earns the fifth point only for an answer of $H(t) = 1 + 3e^{\sin(t/2)}$ or a mathematically equivalent expression for $H(t)$ such as $H(t) = 1 + e^{\sin(t/2)+\ln 3}$.
- A response does not need to argue that $\lvert H-1\rvert = H-1$ in order to earn the fifth point.
- Special case: A response that presents an incorrect separation of variables of $\frac{1}{2}\cdot\frac{dH}{H-1} = \cos\left(\frac{t}{2}\right)dt$ does not earn the first point or the fifth point but is eligible for the 2 antiderivative points. If the response earns at least 1 of the 2 antiderivative points, then the response is eligible for the fourth point.

Total for part (c) **5 points**

Total for question 3 **9 points**