5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

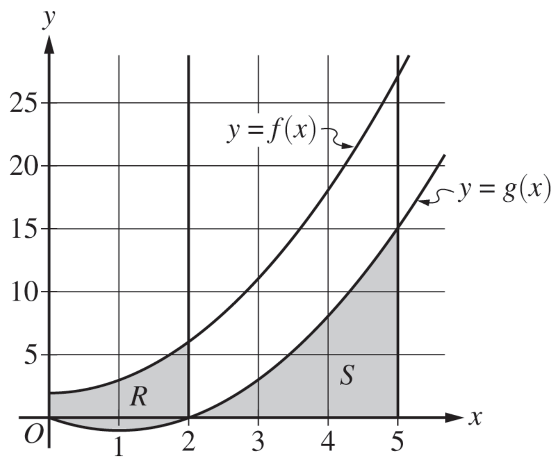
(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.



6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

Model Solution	Scoring	

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

$\left.\frac{dy}{dx}\right\rvert_{(x,\, y)=(2,\, 4)} = \frac{-2(2)}{3 + 4(4)} = -\frac{4}{19}$	Slope of tangent line	**1 point**
$y \approx 4 - \frac{4}{19}(3 - 2) = \frac{72}{19}$	Approximation	**1 point**

Scoring notes:

- A response earns the first point for finding $\left.\frac{dy}{dx}\right\rvert_{(x,\, y)=(2,\, 4)} = -\frac{4}{19}$, even if this is not labeled or used as the slope of a tangent line.
- A response that does not explicitly find the value of $\left.\frac{dy}{dx}\right\rvert_{(x,\, y)=(2,\, 4)}$ but uses a slope of $-\frac{4}{19}$ in a linear approximation also earns the first point.
- A response that declares $\left.\frac{dy}{dx}\right\rvert_{(x,\, y)=(2,\, 4)}$ equal to any nonzero value other than $-\frac{4}{19}$ does not earn the first point but is eligible for the second point for a linear approximation at $x = 3$ through the point $(2, 4)$ with a slope of the declared value.
  - The second point cannot be earned with a linear approximation using a slope other than $-\frac{4}{19}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right\rvert_{(x,\, y)=(2,\, 4)}$.
- The second point cannot be earned for an approximation at any value of x other than 3.
- A response does not have to write the tangent line equation but must clearly demonstrate its use at $x = 3$ in finding the requested approximation in order to earn both points.
- The minimal work required to earn both points is $4 - \frac{4}{19}(3 - 2)$.

Total for part (a) 2 points

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$\frac{dy}{dx} = \frac{-2x}{3+4y} = 0 \Rightarrow x = 0$	Considers $\frac{dy}{dx} = 0$	**1 point**
And so, if the horizontal line $y = 1$ is tangent to the curve, the point of tangency must be $(0, 1)$. However, the point $(0, 1)$ is not on the curve, because $0^2 + 3 \cdot 1 + 2 \cdot 1^2 = 5 \neq 48$. Therefore, the horizontal line $y = 1$ is not tangent to the curve.	Answer with reason	**1 point**

Scoring notes:

- The first point can be earned with a response of $\frac{dy}{dx} = 0,\ -2x = 0,$ or $x = 0,$ OR by identifying the point $(0, 1)$.
- To earn the second point a response must provide a reason that the line $y = 1$ is not tangent to the curve. Merely stating "$(0, 1)$ does not lie on the curve" is not sufficient to earn the second point.
- Alternate solution:

  If $y = 1,$ then $x^2 + 3 \cdot 1 + 2 \cdot 1^2 = 48 \Rightarrow x = \pm\sqrt{43}$.

  $$\left.\frac{dy}{dx}\right|_{(x,\,y)=\left(\pm\sqrt{43},\,1\right)} = \frac{\pm 2\sqrt{43}}{7} \neq 0$$

  Therefore, the horizontal line $y = 1$ is not tangent to the curve.

  - A response that uses this method earns the first point by using $y = 1$ in $x^2 + 3y + 2y^2 = 48$.
  - A response that fails to consider both $x = +\sqrt{43}$ and $x = -\sqrt{43}$ does not earn the second point.

Total for part (b) 2 points

(c) The curve intersects the positive x-axis at the point $\left(\sqrt{48}, 0\right)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

At the point $\left(\sqrt{48}, 0\right)$, the slope of the line tangent to the curve is $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3 + 4(0)}$. The denominator of $\frac{dy}{dx}$ is $3 + 4(0)$, which does not equal 0. Therefore, the line tangent to the curve at this point is not vertical.	Answer with reason	**1 point**

Scoring notes:

- A response does not need to consider the numerator of $\left.\frac{dy}{dx}\right|_{(x,\, y)=\left(\sqrt{48},\, 0\right)}$ in order to earn this point; considering the denominator is sufficient.
- To earn this point a response must clearly demonstrate that the slope of the tangent line at the point $\left(\sqrt{48}, 0\right)$ is defined and answer "no."
  - Such demonstrations include, but are not limited to, the following:
    - $3 + 4(0) \neq 0$
    - At $\left(\sqrt{48}, 0\right)$, $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3}$.
    - $\frac{-2\sqrt{48}}{3}$
    - When $3 + 4y = 0$, $y \neq 0$.

Total for part (c) 1 point

(d) For time $t \geq 0,$ a particle is moving along another curve defined by the equation $y^3 + 2xy = 24.$ At the instant the particle is at the point $(4, 2),$ the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

$3y^2\dfrac{dy}{dt} + 2x\dfrac{dy}{dt} + 2y\dfrac{dx}{dt} = 0$	Attempts implicit differentiation	**1 point**
	$3y^2\dfrac{dy}{dt} + 2x\dfrac{dy}{dt} + 2y\dfrac{dx}{dt} = 0$	**1 point**
$\dfrac{dy}{dt} = -2$	Uses $\dfrac{dy}{dt} = \pm 2$	**1 point**
$3(2)^2(-2) + 2(4)(-2) + 2(2)\dfrac{dx}{dt} = 0 \Rightarrow \dfrac{dx}{dt} = \dfrac{40}{4} = 10$ The rate of change with respect to time in the x-coordinate is 10 units per second.	Answer	**1 point**

Scoring notes:

- The first point is earned for implicitly differentiating $y^3 + 2xy = 24$ with respect to t with at most one error.
  - The first point can also be earned by correctly differentiating with respect to x: $3y^2\dfrac{dy}{dx} + 2x\dfrac{dy}{dx} + 2y = 0.$
- The second point is earned for an equation equivalent to $3y^2\dfrac{dy}{dt} + 2x\dfrac{dy}{dt} + 2y\dfrac{dx}{dt} = 0.$
- A response does not need to explicitly declare $\dfrac{dy}{dt} = -2$ or $\dfrac{dy}{dt} = 2$ in order to earn the third point; this point may be earned by correctly substituting $\dfrac{dy}{dt} = -2$ or $\dfrac{dy}{dt} = 2$ in the implicitly differentiated equation. However, a response that uses both $\dfrac{dy}{dt} = -2$ and $\dfrac{dy}{dt} = 2$ in the implicitly differentiated equation does not earn the third point.
- The fourth point cannot be earned without the first 3 points. The fourth point is earned only for the value of 10 with no mistakes in supporting work.
  - Note that a response that uses $\dfrac{dy}{dt} = 2$ and then mishandles subtracting 40 from both sides of the equation, e.g., $3(2)^2(2) + 2(4)(2) + 2(2)\dfrac{dx}{dt} = 0 \Rightarrow \dfrac{dx}{dt} = \dfrac{40}{4},$ does not earn the fourth point.

Total for part (d) 4 points

Total for question 5 9 points