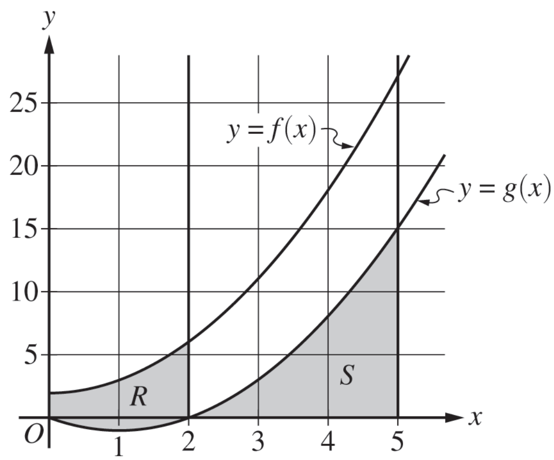


6. The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

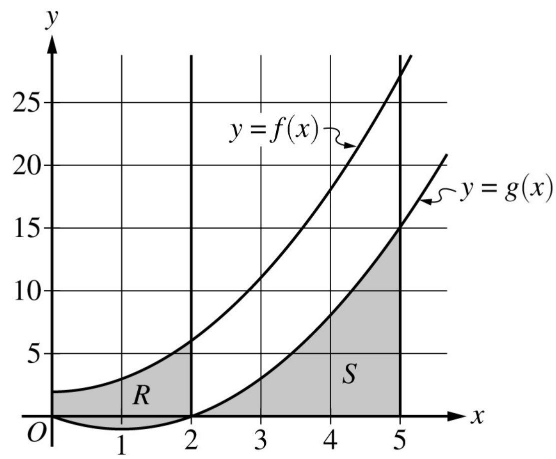
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

	Model Solution	Scoring	
(a)	Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.		
	Area $= \int_0^2 (f(x) - g(x))\, dx$	Integrand	**1 point**
		Answer	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $f(x) - g(x)$, $|f(x) - g(x)|$, $g(x) - f(x)$, or $|g(x) - f(x)|$ in one or more definite integrals.
- The first point could also be earned for a difference of definite integrals with integrands $f(x)$ and $g(x)$.
- The second point is earned only for one or more integrals equivalent to $\int_0^2 (f(x) - g(x))\, dx$, such as $\int_0^2 f(x)\, dx - \int_0^2 g(x)\, dx$, $\int_0^2 |f(x) - g(x)|\, dx$, $-\int_0^2 (g(x) - f(x))\, dx$, or $\int_0^2 |g(x) - f(x)|\, dx$.
  - Note: $\int_0^2 f(x)\, dx + \left|\int_0^2 g(x)\, dx\right|$ would earn both points.

Total for part (a) — 2 points

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

$\text{Volume} = \int_2^5 \frac{1}{2}(g(x))^2\,dx = \int_2^5 \frac{1}{2}\left(x^2 - 2x\right)^2\,dx$	Integrand	**1 point**
	Limits	**1 point**
$= \frac{1}{2}\int_2^5 \left(x^4 - 4x^3 + 4x^2\right)dx$ $= \frac{1}{2}\left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_2^5$	Antiderivative	**1 point**
$= \frac{1}{2}\left[\left(\frac{5^5}{5} - 5^4 + \frac{500}{3}\right) - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)\right]$ $= \frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right) = \frac{414}{5}$	Answer	**1 point**

Scoring notes:

- The first point is earned only by a response with an integrand of the form $k(g(x))^2$ in a definite integral, where k is any nonzero constant.
- The second point is earned for limits of $x = 2$ and $x = 5$ in a definite integral with an integrand of the form $a(x) \cdot g(x)$ for any nonzero factor $a(x)$.
- To earn the third point a response must provide a correct antiderivative of $k\left(x^2 - 2x\right)^n$ for some integer $n \geq 2$.
- The fourth point is earned only for a numeric answer equivalent to $\frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right)$.
- Special case: A response of $\text{Volume} = \int_2^5 \frac{1}{2}(f(x))^2\,dx$ or $\text{Volume} = \int_2^5 \frac{1}{2}\left(x^2 + 2\right)^2\,dx$ earns the first 2 points and is not eligible for the last 2 points.

Total for part (b) 4 points

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

$\text{Volume} = \pi\int_2^5 \left[\left(20^2\right) - (20 - g(x))^2\right] dx$ $= \pi\int_2^5 \left[400 - (20 - g(x))^2\right] dx$ $= \pi\int_2^5 \left[400 - \left(20 - \left(x^2 - 2x\right)\right)^2\right] dx$	Form of integrand	**1 point**
	Integrand	**1 point**
	Limits and constant	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $20^2 - (20 - g(x))^2$, $20^2 - (g(x) - 20)^2$, $(20 - g(x))^2 - 20^2$, $(g(x) - 20)^2 - 20^2$, or any mathematically equivalent expression, in one or more definite integrals.
- The second point is earned only for an integrand mathematically equivalent to $20^2 - (20 - g(x))^2$ or $20^2 - (g(x) - 20)^2$ in a definite integral. Note that $\int_a^b \left|(20 - g(x))^2 - 400\right| dx$ or $\left|\int_a^b \left((20 - g(x))^2 - 400\right) dx\right|$ earns the first 2 points.
- The integral may be split into two integrals.
  - For example, $\text{Volume} = \pi\int_2^5 \left(20^2\right) dx - \pi\int_2^5 (20 - g(x))^2\, dx$ or $\text{Volume} = \pi \cdot 20^2 \cdot 3 - \pi\int_2^5 (20 - g(x))^2\, dx.$
- A response that presents an allowable integrand involving $g(x)$, but continues and makes an error in using the expression for $g(x)$, does not earn the second point.
  - For example, $\pi\int_2^5 \left[20^2 - (20 - g(x))^2\right] dx = \pi\int_2^5 \left[20^2 - \left(20 - x^2 - 2x\right)^2\right] dx$ earns the first point but does not earn the second point.
- To be eligible for the third point a response must have earned at least 1 of the first 2 points or must have presented an integrand involving $g(x)$ of the form $R^2 - r^2$ in a definite integral.
- The third point is earned only by a definite integral including the constant π and limits $x = 2$ to $x = 5$. A response that presents any other constant or limits (including $x = 5$ to $x = 2$, except in the note below) does not earn the third point.
  - Note: $\pi\int_5^2 \left[(20 - g(x))^2 - 400\right] dx$ would earn all three points.
- Special case: A response of $\pi\int_2^5 \left[400 - (20 - f(x))^2\right] dx$ or equivalent earns 2 of the 3 points.

Total for part (c) 3 points

Total for question 6 9 points