3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

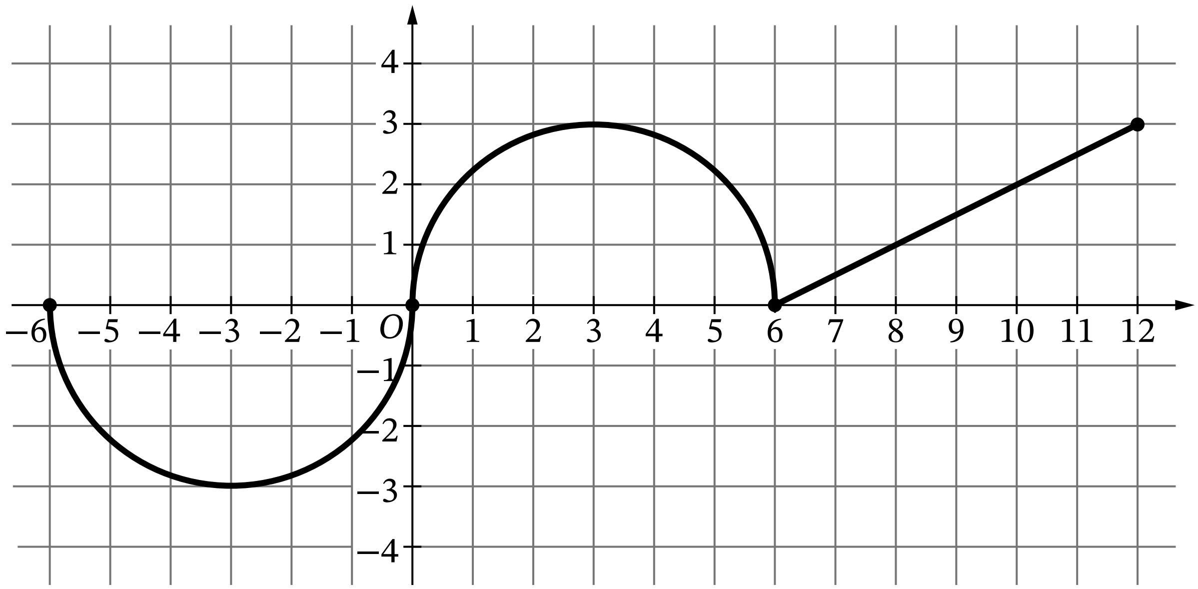
A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

4. The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

Part B (AB or BC): Graphing calculator not allowed

Question 3 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

	Model Solution	Scoring

A Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

Model Solution	Scoring	
$R'(1) \approx \dfrac{R(2) - R(0)}{2 - 0}$	Answer with setup	**Point 1 (P1)**
$= \dfrac{100 - 90}{2} = \dfrac{10}{2} = 5$ words per minute per minute	Units	**Point 2 (P2)**

Scoring Notes for Part A

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
  - $\dfrac{100 - 90}{2 - 0}$, $\dfrac{10}{2 - 0}$, $\dfrac{100 - 90}{2}$, or $\dfrac{R(2) - R(0)}{2 - 0} = 5$ is sufficient to earn **P1**.
  - $\dfrac{R(2) - R(0)}{2 - 0}$ by itself is not sufficient to earn **P1**.
- **P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- **P2** is also earned for the units "$\text{words}/\text{minute}^2$."

B	Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.		
	R is differentiable implies R is continuous.	Differentiable implies continuous	**Point 3 (P3)**
	$R(0) = 90 < 155 < R(10) = 162$ Therefore, by the Intermediate Value Theorem, there must be a value c, with $0 < c < 10$, such that $R(c) = 155$.	Answer with justification	**Point 4 (P4)**

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states "R is continuous" without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that "R is continuous," and answer "yes" in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

C	Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.		
	$\int_0^{10} R(t)\,dt$ $\approx \frac{R(0)+R(2)}{2}(2-0)+\frac{R(2)+R(8)}{2}(8-2)$ $+\frac{R(8)+R(10)}{2}(10-8)$	Form of trapezoidal sum	**Point 5 (P5)**
	$=\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ $=\frac{190}{2}(2)+\frac{250}{2}(6)+\frac{312}{2}(2)=190+750+312=1252$	Answer with supporting work	**Point 6 (P6)**

Scoring Notes for Part C

- Read "=" as "$\approx$" for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
  - $\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ earns P5 and is sufficient to earn **P6**.
  - $\frac{190}{2}(2)+\frac{250}{2}(6)+\frac{312}{2}(2)$ earns P5 and is sufficient to earn **P6**.
  - $\frac{1}{2}((R(0)+R(2))(2)+(R(2)+R(8))(6)+(R(8)+R(10))(2))$ earns **P5** and is eligible for **P6**.
  - $\frac{90+100}{2}(2)+\frac{100+150}{2}(2)+\frac{150+162}{2}(2)$ earns P5 but is not eligible for **P6**.
    (Note that the factor of 2 in the second term of this expression is incorrect.)
- **Special case:** A response of $(90+100)+(100+150)3+(150+162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.
  Special case: A response of $95 \cdot 2 + 125 \cdot 6 + 156 \cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90+100}{2}(2-0)+\frac{100+150}{2}(8-2)+\frac{150+162}{2}(10-8)$ or equivalent banks **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
- A response of $\frac{(90\cdot 2+100\cdot 6+150\cdot 2)+(100\cdot 2+150\cdot 6+162\cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90\cdot 2+100\cdot 6+150\cdot 2=1080$) or a completely correct right Riemann sum (e.g., $100\cdot 2+150\cdot 6+162\cdot 2=1424$) earns **P5** but does not earn **P6**.

D A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

$\int_0^{10} W(t)\,dt = \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100\right) dt$	Integrand	**Point 7 (P7)**
$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t\right)\Big\vert_0^{10}$	Antiderivative	**Point 8 (P8)**
$= \left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ $= 1300$ Based on the model, the teacher has read 1300 words by the end of the 10 minutes.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt.
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

C It can be shown that $v_J'(2) > 0$. Is the speed of particle *J* increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$v_J(2) > 0$ and $v_J'(2) > 0$. Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle *J* is increasing at $t = 2$.	Answer with reason	**Point 6 (P6)**

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated "$v_J(t) > 0$ for $1 < t < 5$" in part B does not need to restate $v_J(2) > 0$ and earns **P6** for "$v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing."

D Particle *J* is at position $x = 7$ at time $t = 0$. Find the position of particle *J* at time $t = 2$. Show the work that leads to your answer.

$x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3\,dt$	Integrand	**Point 7 (P7)**
$= 7 + \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2$	Antiderivative	**Point 8 (P8)**
$= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t\left(t^2 - 1\right)^3$. (See below for notes on how to handle a missing differential dt.)
- **P8** is earned for an antiderivative of the form $k\left(t^2 - 1\right)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

  Note: An ambiguous response, such as $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right.$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:
  - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2 + 7$ resolves the ambiguity.

      Therefore, this earns **P7** and **P8** and is eligible for **P9**.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4 + 7t\right]_0^2$ confirms that an incorrect integrand was used.

      Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
    - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using u-substitution:

  Let $u = t^2 - 1$, then $du = 2t\ dt$.

  $t = 0 \Rightarrow u = -1$

  $t = 2 \Rightarrow u = 3$

  $x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3 dt$

  $= 7 + \int_{-1}^3 u^3\,du = 7 + \left[\frac{1}{4}u^4\right]_{-1}^3$

  $= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$
- Alternate solution using indefinite integral:

  $\int 2t\left(t^2 - 1\right)^3 dt = \frac{1}{4}\left(t^2 - 1\right)^4 + C$

  $x_J(0) = 7 = \frac{1}{4}\left(0^2 - 1\right)^4 + C \Rightarrow C = \frac{27}{4}$

  $x_J(t) = \frac{1}{4}\left(t^2 - 1\right)^4 + \frac{27}{4}$

  $x_J(2) = \frac{1}{4}\left(2^2 - 1\right)^4 + \frac{27}{4} = \frac{108}{4} = 27$