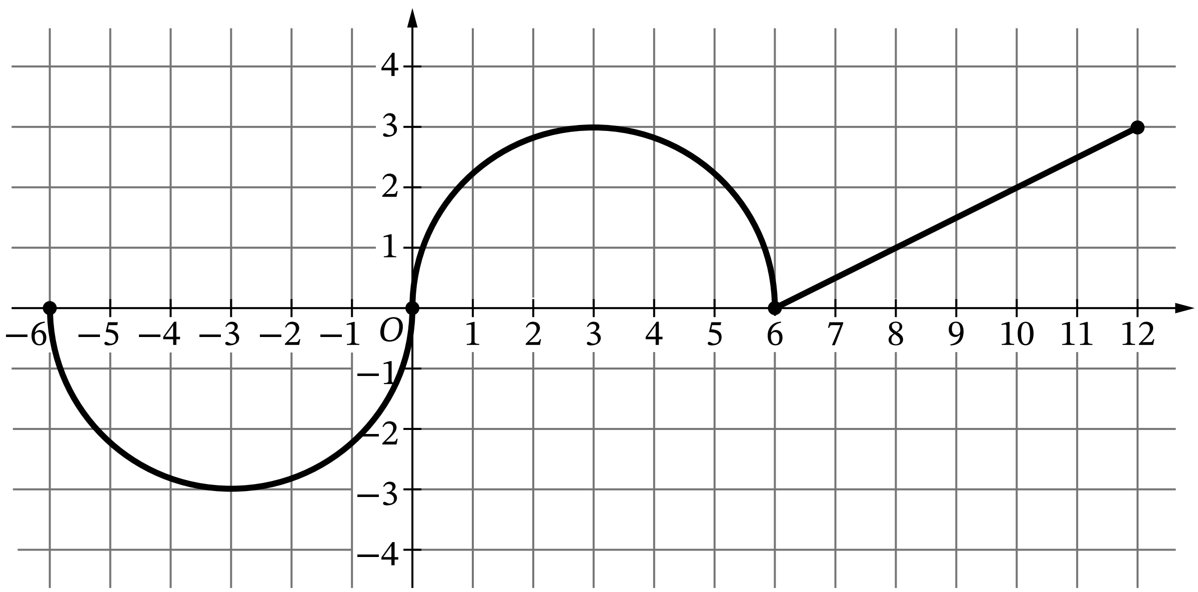
4. The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

5. Two particles, *H* and *J*, are moving along the *x*-axis. For $0 \le t \le 5$, the position of particle *H* at time *t* is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle *J* at time *t* is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle *H* at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time *t*, for $0 < t < 5$, are particles *H* and *J* moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J'(2) > 0$. Is the speed of particle *J* increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle *J* is at position $x = 7$ at time $t = 0$. Find the position of particle *J* at time $t = 2$. Show the work that leads to your answer.

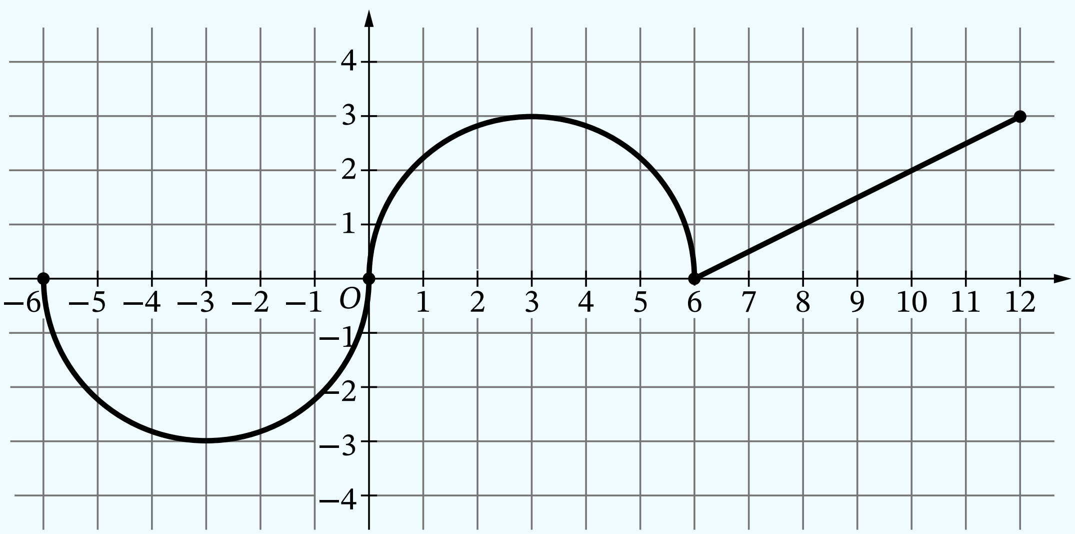
Part A (AB or BC): Graphing calculator not allowed

Question 4 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\, dt$.

	Model Solution	Scoring	
A	Find $g'(8)$. Give a reason for your answer.		
	$g'(x) = f(x)$	Considers $g'(x) = f(x)$	**Point 1 (P1)**
	$g'(8) = f(8) = 1$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$).
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

B Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa.	Answer	**Point 3 (P3)**
The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there.	Reason	**Point 4 (P4)**
The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.		

Scoring Notes for Part B

- **P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there" earns both **P3** and **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there" earns **P3** but not **P4**.
  - A response of "g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there" earns **P3** but not **P4**.
  - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.
- **Special case:** A response with two of the three correct x-values with correct reasoning and no other/additional values of x declared to be points of inflection earns **P4** but not **P3**.

C	Find $g(12)$ and $g(0)$. Label your answers.		
	$g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}\cdot 6\cdot 3 = 9$	$g(12)$	**Point 5 (P5)**
	$g(0) = \int_6^{0} f(x)\,dx = -\int_0^{6} f(x)\,dx = -\frac{\pi}{2}3^2 = -\frac{9\pi}{2}$	$g(0)$	**Point 6 (P6)**

Scoring Notes for Part C

- Unlabeled values do not earn either **P5** or **P6**.
- **P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- **P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

  Note: Incorrect communication between the label "$g(0)$" and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^{6} f(x)\,dx = -\frac{9\pi}{2}$ earns **P6**.

D	Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.		
	For $-6 \le x \le 12$, g attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint. $g'(x) = f(x) = 0$	Considers $g'(x) = 0$	**Point 7 (P7)**
	$\Rightarrow \ x = 0, \ x = 6$ x | $g(x)$ -6 | 0 0 | $-\frac{9\pi}{2}$ 6 | 0 12 | 9	Justification	**Point 8 (P8)**
	Therefore, on the closed interval $-6 \le x \le 12$, g attains an absolute minimum value at $x = 0$.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.

  A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase "critical points of g" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x-values).
- Alternate justification and answer:

  Because $g'(x) \le 0$ (or $f(x) \le 0$) for $-6 \le x < 0$ and $g'(x) \ge 0$ (or $f(x) \ge 0$) for $0 < x \le 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.