6. Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$.

B. There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

C. For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.

D. A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

STOP

END OF EXAM

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

	Model Solution	Scoring	
A	Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.		
	$\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $\Rightarrow\ 3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$	Implicit differentiation	**Point 1 (P1)**
	$\Rightarrow\ (3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ $\Rightarrow\ \frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$	Verification	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- To be eligible for **P2**, a response must have earned **P1**.
- It is sufficient to present $(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

B	There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.		
	$\left.\frac{dy}{dx}\right\|_{(x,\ y)=(2,\ -1)} = \frac{-2}{2(3+2-1)} = -\frac{1}{4}$	Slope of tangent line	**Point 3 (P3)**
	$y \approx -1 - \frac{1}{4}(1.6-2) = -0.9$	Tangent line approximation	**Point 4 (P4)**

Scoring Notes for Part B

- A response can earn **P3** with $\left.\frac{dy}{dx}\right|_{(x,\ y)=(2,\ -1)} = -\frac{1}{4}$, $\frac{dy}{dx} = -\frac{1}{4}$, "slope is $-\frac{1}{4}$," or equivalent.
- A response that presents a linear approximation with a slope of $-\frac{1}{4}$ also earns **P3**.
- A response that declares $\left.\frac{dy}{dx}\right|_{(x,\ y)=(2,\ -1)}$ (or the slope) equal to any nonzero value $k \neq -\frac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.
- **P4** cannot be earned with a linear approximation using a slope other than $-\frac{1}{4}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right|_{(x,\ y)=(2,\ -1)}$.
- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation to be eligible for **P4**.
- A response of $-1 - \frac{1}{4}(-0.4)$ earns both **P3** and **P4**.
- A response of $-1 - \frac{1}{4}(1.6 - 2)$ or equivalent banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4)**.

  Note: An ambiguous response, such as $-1 - \frac{1}{4}(1.6 - 2$, does not bank **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

C	For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.		
	For $x > 0$, the curve G has a vertical tangent line when $2\left(3y^2 - 2y - 1\right) = 0$.	Sets denominator equal to 0	**Point 5 (P5)**
	$2\left(3y^2 - 2y - 1\right) = 0 \;\Rightarrow\; 2(3y + 1)(y - 1) = 0$ Because $y > 0$, it follows that $y = 1$. The line tangent to the curve is vertical at the point on the curve where $y = 1$.	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned with any of $2\left(3y^2 - 2y - 1\right) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.
- To be eligible for **P6**, a response must have earned **P5**.
- A response does not need to consider the numerator of $\frac{dy}{dx} = \frac{-x}{2\left(3y^2 - 2y - 1\right)}$ to earn **P5** or **P6**; considering the denominator is sufficient.
- A response that states solutions of $y = -\frac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.
- In the presence of algebraic work to find the value of y, **P6** is earned only if the algebraic work is correct.
- A response of "$2\left(3y^2 - 2y - 1\right) = 0$, $y = 1$" earns both **P5** and **P6**.

D	A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.		
	$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$ $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	Attempts implicit differentiation with respect to t	**Point 7 (P7)**
		$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	**Point 8 (P8)**
	$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$ $\Rightarrow\ 6 + 9\frac{dy}{dt} = 0 \ \Rightarrow\ \frac{dy}{dt} = -\frac{2}{3}$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.
- **P8** is earned for an equation equivalent to $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$.
- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.
- **P9** is earned only for the value of $-\frac{2}{3}$.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$

$$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \ \Rightarrow\ 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \ \Rightarrow\ \frac{dy}{dx} = \frac{-2y}{2x + \frac{1}{y}}$$

$$\left.\frac{dy}{dx}\right|_{(x,\, y)=(4,\, 1)} = \frac{-2(1)}{2(4) + \frac{1}{1}} = -\frac{2}{9}$$

$$\left.\frac{dy}{dt}\right|_{(x,\, y)=(4,\, 1)} = \left.\frac{dy}{dx}\cdot\frac{dx}{dt}\right|_{(x,\, y)=(4,\, 1)} = -\frac{2}{9}\cdot 3 = -\frac{2}{3}$$

  - **P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\frac{dy}{dx}$ is eventually correctly linked to $\frac{dx}{dt}$.
  - **P8** is earned for finding $\frac{dy}{dx}$ and multiplying the result by $\frac{dx}{dt} = 3$.
  - **P8** can be earned for a stated incorrect $\frac{dy}{dx}$, as long as it is multiplied by 3.
  - **P9** is only earned for a correct value of $-\frac{2}{3}$ or equivalent.
  - Stating $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ alone does not earn any points.