2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

   The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

   (A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

   (ii) Find the values for a and b as decimal approximations.

   (B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   (ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

   (iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

   (C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

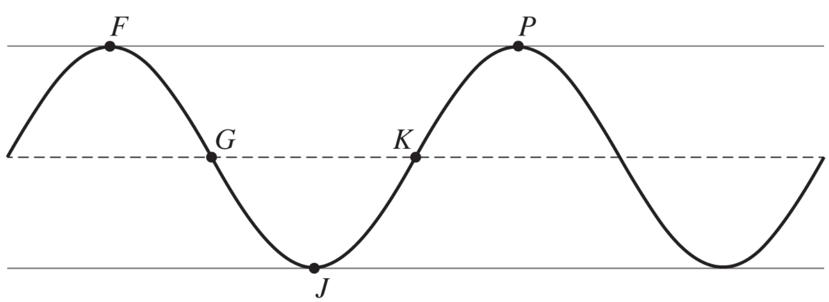
Note: Figure not drawn to scale.

3. The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

Question 2: Modeling a Non-Periodic Context
Part A: Graphing calculator required — 6 points

On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

Model Solution	Scoring	

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

Model Solution	Scoring	
(i) Because $G(0) = 40$ and $G(91) = 76$, two equations to find a and b are $a + b\ln(0+1) = 40$ $a + b\ln(91+1) = 76$.	Two equations	**1 point**
(ii) $a = 40 - b\ln 1 = 40$ $b = \dfrac{(76-40)}{\ln 92} = 7.961451$ $G(t) = 40 + 7.961\ln(t+1)$	Values of a and b	**1 point**

General Scoring Notes for Question 2 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for presenting two equations involving a and b that use the given input-output pairs.
- The second point is earned for correct values of a and b with or without supporting work. If correct values are identified, work should be ignored.
- The second point is earned for correct values of a and b presented as either stand-alone values OR in an expression for $G(t)$.
- A response is eligible to earn both points with a correct translation to “thousands.” Use of $40,000$ and $76,000$ results in values of $a = 40,000$ and $b = 7961$.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one correct equation in the presence of two equations involving a and b AND one correct value. Partial credit response is scored **1-0** in Part (A).

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(i) $\dfrac{G(91) - G(0)}{91 - 0} = \dfrac{(76 - 40)}{91} = 0.395604$ The average rate of change is 0.396 (or 0.395) thousand units per day.	Average rate of change	**1 point**
(ii) The average rate of change is $r = \dfrac{G(91) - G(0)}{91 - 0} = 0.395604$. The secant line between point $(0, G(0))$ and point $(91, G(91))$ is given by $y = y_1 + r(x - x_1)$, where (x_1, y_1) can be either one of the points. Estimates using the average rate of change are given by $y = G(0) + r(t - 0)$ OR $y = G(91) + r(t - 91)$. Both of these produce the same estimate. For $t = 50$, $y = 40 + r(50 - 0) = 59.780$ OR $y = 76 + r(50 - 91) = 59.780$. The number of units sold on day $t = 50$ was approximately 59.780 thousand.	Estimate using average rate of change	**1 point**
(iii) The estimate A_t is the y-coordinate of a point on the secant line that passes through $(0, G(0))$ and $(91, G(91))$. Because the graph of G is concave down on the interval $(0, 91)$, this secant line is below the graph of G on the interval $(0, 91)$. Therefore, the estimate A_t is less than the value of $G(t)$ for all t on the interval $(0, 91)$.	Answer with explanation	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii).
- The first point is earned for a correct decimal approximation in the presence of a quotient that uses the given data values. Units are not needed and are ignored if presented.
- Eligibility for the second point:
  - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59.750, 59.805]$ earns the second point in the presence of supporting work.
  - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
  - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- The final number in (ii) may be reported as 59 thousand or 60 thousand provided the supporting work has a correct decimal approximation for the estimate.
- A response is eligible to earn both points with a correct translation to "thousands."
  - Use of $40,000$ and $76,000$ results in an answer of 395.604 in (i).
  - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59,750,\ 59,805]$ earns the second point in the presence of supporting work.
  - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
  - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- A response that does not earn either point in Part (B) (i) and Part (B) (ii) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B)(i)/(ii).

First Column	**Second Column**
A correct quotient that uses the given data values that is not expressed as a decimal approximation	A correct estimate in (ii) that does not include supporting work
A correct quotient that uses the given data values and has a decimal presentation error	Correct supporting work in (ii) that does not provide an estimate
A correct average rate of change in (i) that does not include supporting work	

- To earn the third point, the reasoning must include:
  - The graph of G is concave down OR the rate of change of G is decreasing
  - A reference to the use of a secant line on $0 < t < 91$ OR the use of a linear function with reference to endpoints 0 and 91 that provide the placement of the line

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

On day $t = 91$, the output for daily sales and $G(91)$ are the same. For $t > 91$, daily sales are decreasing and G is increasing. Therefore, the absolute value of the difference between the actual daily sales and the daily sales predicted by G is increasing each day for $t > 91$.	Answer with reason	**1 point**

Scoring notes:

- To earn the point, the reasoning must include an implicit or explicit connection between the “function model is increasing” and “daily sales are decreasing.”

Total for question 2 **6 points**