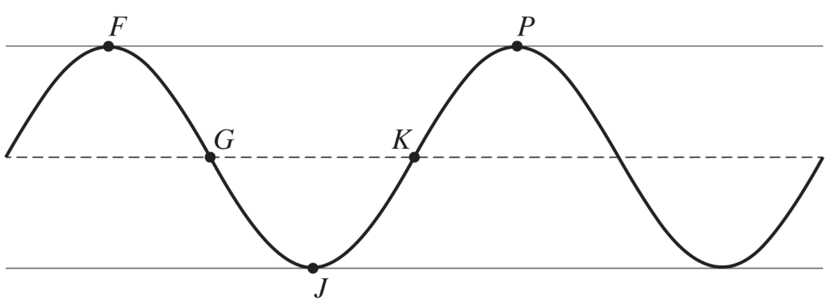
Note: Figure not drawn to scale.

3. The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Question 3: Modeling a Periodic Context
Part B: Graphing calculator not allowed **6 points**

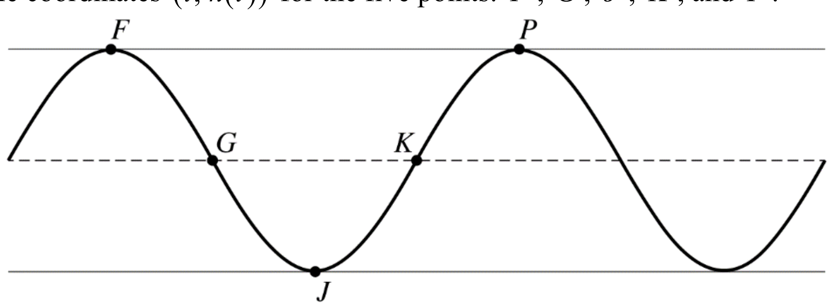
Note: Figure not drawn to scale.

The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

Model Solution	Scoring

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.

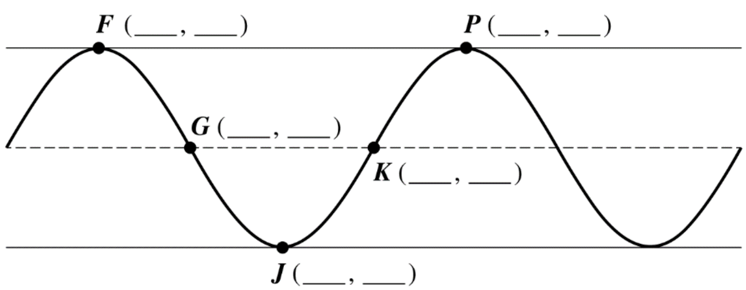


F has coordinates $\left(\frac{3}{2}, 18\right)$.	$h(t)$-coordinates	**1 point**
G has coordinates $(2, 9)$.	t-coordinates	**1 point**
J has coordinates $\left(\frac{5}{2}, 0\right)$.		
K has coordinates $(3, 9)$.		
P has coordinates $\left(\frac{7}{2}, 18\right)$.		
OR		

F has coordinates $\left(-\frac{1}{2}, 18\right)$. G has coordinates $(0, 9)$. J has coordinates $\left(\frac{1}{2}, 0\right)$. K has coordinates $(1, 9)$. P has coordinates $\left(\frac{3}{2}, 18\right)$. Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.	

Scoring notes:

- No supporting work is required.
- $h(t)$-coordinates and/or t-coordinates may appear in a list. Negative t-coordinates are acceptable.
- t-coordinates must be $-\frac{1}{2} + 2k, 0 + 2k, \frac{1}{2} + 2k, 1 + 2k, \frac{3}{2} + 2k$, for a specific integer k.
- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.



- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response meets one of the following criteria. Partial credit response is scored **0-1** in Part (A).
  - All 5 points in the form $(h(t), t)$ with correct input values and correct output values swapped
  - 3 correct points out of the 5 points
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that
      (1) F and P have same $h(t)$-coordinate
      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P
      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

(B) The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

$h(t) = a\sin(b(t+c)) + d$ $a = 9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = -1$ $d = 9$ $h(t) = 9\sin(\pi(t-1)) + 9$ OR $a = -9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = 0$ $d = 9$ $h(t) = -9\sin(\pi t) + 9$ Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.	Vertical transformations: Values of a and d Horizontal transformations: Values of b and c	**1 point** **1 point**

Scoring notes:

- No supporting work is required.
- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.
- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.

$a =$ ______________
$b =$ ______________
$c =$ ______________
$d =$ ______________

- Other correct values of c:
  - $h(t) = 9\sin(\pi(t+c)) + 9 \Rightarrow c = -1 + 2k$, for any integer k
  - $h(t) = -9\sin(\pi(t+c)) + 9 \Rightarrow c = 0 + 2k$, for any integer k
- Full credit for Part (B) is possible based on the correct use of an imported response from Part (A) that meets these criteria:
  - $a \neq 1$, $b \neq 1$, $d \neq 0$, and if $a > 0$, then $c \neq 0$
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that
      (1) F and P have same $h(t)$-coordinate
      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P
      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response meets one of the following criteria. Partial credit response is scored **1-0** in Part (B).
  - Values of a and b [Values of a and b could be $\pm$]
  - Values of b and d [Value of b could be $\pm$]
  - Response uses $h(t) = a\mathbf{cos}(b(t+c)) + d$ with values as follows:
    - $a = 9$; $b = \pi$; $c = -\frac{3}{2} + 2k$, for a specific integer k; $d = 9$
    - $a = -9$; $b = \pi$; $c = -\frac{1}{2} + 2k$, for a specific integer k; $d = 9$

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.
b. h is positive and decreasing.
c. h is negative and increasing.
d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

(i) Choice a.	Function behavior	**1 point**
(ii) Because the graph of h is concave down on the interval (t_1, t_2), the rate of change of h is decreasing on the interval (t_1, t_2).	Change in rate of change	**1 point**

Scoring notes:

- No supporting work is required.
- The first point is earned for a correct answer of "a" OR "positive and increasing." If both the letter choice and written description are included, the written description is scored.
- To earn the second point, "decreasing" OR "function h is increasing at a decreasing rate" is acceptable. If concavity of the graph of h is referenced, it must be correct.
- The second point is not earned for a response that only includes "the graph of h is concave down."
- A response with a statement that the rate of change of h is decreasing at an increasing (or decreasing) rate does not earn the second point. Analysis to make such a conclusion requires calculus.
- The second point is not earned for a response that states "increasing at a decreasing rate" without a subject. The implied subject is "the rate of change of h."
- The second point cannot be earned if there are any errors in Part (C) (ii).

Total for question 3 **6 points**