4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed — 6 points

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

Model Solution	Scoring	

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

Model Solution	Scoring	
(i) $g(x) = 10$ $e^{(x+3)} = 10$ $\ln e^{(x+3)} = \ln 10$ $x + 3 = \ln 10$ $x = -3 + \ln 10$	Solution to $g(x) = 10$	**1 point**
(ii) $h(x) = \frac{\pi}{4}$ $\arcsin\left(\frac{x}{2}\right) = \frac{\pi}{4}$ $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ $\frac{x}{2} = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$) $x = \sqrt{2}$	Solution to $h(x) = \frac{\pi}{4}$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). "Scratchwork" can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a correct solution may be considered scratchwork.
- Supporting work in (ii) must include $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ OR $\frac{x}{2} = \frac{\sqrt{2}}{2}$ OR $\frac{x}{2} = \frac{1}{\sqrt{2}}$.
- An alternate solution for (i) is $x = \frac{\log_b 10}{\log_b e} - 3$, where $b > 0$, $b \neq 1$, and the result is evaluated according to bullets 2 and 3 in the directions.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point. Rationalizing denominators is not required.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (A).

First Column	**Second Column**
Correct answer in (i) without supporting work	Correct answer in (ii) without supporting work
Correct answer in (i) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.	Correct answer in (ii) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.
Answer in (i) is reported as $x + 3 = \ln 10$. No incorrect work follows.	Answer in (ii) is reported as $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{\sqrt{2}}{2}$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{1}{\sqrt{2}}$. No incorrect work follows.

(B) The functions j and k are given by

$$j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x .$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(i) $j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$ $j(x) = \log_{10}\left(8x^5 \cdot 2x^2\right) - \log_{10} x^9$ $j(x) = \log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16x^7}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16}{x^2}\right),\ x > 0$	Expression for $j(x)$	**1 point**
(ii) $k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x$ $k(x) = \left(\frac{\cos^2 x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ $k(x) = \left(\frac{\cos x}{\sin x}\right)$ $k(x) = \frac{1}{\tan x},\ \sin x \neq 0,\ \cos x \neq 0$	Expression for $k(x)$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). “Scratchwork” can be ignored; the use of a variable other than x is acceptable.
- Domain restrictions are not required to be included and are not scored.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point.
- To earn the first point, use of “ $\log$ ” rather than “ $\log_{10}$ ” is acceptable.
- The expression $j(x) = \log_{10}\left(\frac{4}{x}\right)^2$ earns the point in (i) with supporting work.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn the point.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	**Second Column**
Correct expression in (i) without supporting work	Correct expression in (ii) without supporting work
Expression in (i) is reported as $\log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$. No incorrect work follows.	Expression in (ii) is reported as $\frac{\cos x}{\sin x}$ OR $\cot x$. No incorrect work follows.
Expression in (i) is reported as $\log_{10}\left(\frac{16x^7}{x^9}\right)$. No incorrect work follows.	Expression in (ii) includes a correct application of a Pythagorean identity with no incorrect work.
Expression in (i) is reported as $2\log_{10}\left(\frac{4}{x}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-\log_{10}\left(\frac{x^2}{16}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-2\log_{10}\left(\frac{x}{4}\right)$. No incorrect work follows.	
The expression in (i) is reported using natural logarithm and has the correct argument OR any of the expressions in partial credit rows two through six above are presented with natural logarithm.	

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow \cos^{-1}(\tan(2x)) = 0$ $\tan(2x) = \cos(0)$	One value of x	**1 point**
$\tan(2x) = 1$ $2x = \frac{\pi}{4} + \pi n$ $x = \frac{\pi}{8} + \frac{\pi}{2}n$, where n is any integer	All values of x	**1 point**

Scoring notes:

- Supporting work is required. "Scratchwork" can be ignored; the use of a variable other than x is acceptable.
- A response with supporting work that gives all correct values for x, such as $x = \frac{\pi}{8} + \pi n$ and $x = \frac{5\pi}{8} + \pi n$, earns both points.
- When expressing a general solution for all values for x (e.g., $x = \frac{\pi}{8} + \frac{\pi}{2}n$), the response can use i, k, n, or any letter except x, which is the variable used in the function.
- To earn the second point, "where n is any integer" is not required to be included.
- To earn the second point, no incorrect values for x are included.

Total for question 4 **6 points**