1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.
Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

Question 1: Function Concepts
Part A: Graphing calculator required — 6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

	Model Solution	Scoring	
A	**(i)** The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer. **(ii)** Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.		
	(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value	**Point A1**
	(ii) From the table, $f^{-1}(3.5) = 0$.	Value	**Point A2**

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of "$f(1)$" OR "$g(1.75)$" OR "1.75."
- **Point A2** does not require supporting work. **Point A2** is earned with a response of "0."

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	Second Column
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

B

(i) Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233$, $x = 1.578$, $x = 5.643$	Values	**Point B1**
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim\limits_{x\to\infty} g(x) = -\infty$.	End behavior with limit notation	**Point B2**

Scoring Notes for Part B

- **Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233, 1.578, and 5.643. The use of "$x =$" is not required.
- **Point B2** requires a correct limit statement with four components: "lim", "$x \to \infty$", the function g, and $-\infty$. Examples that earn **Point B2** include:
  - $\lim\limits_{x\to\infty} g(x) = -\infty$ OR $\lim\limits_{x\to\infty} g = -\infty$
  - $\lim\limits_{x\to\infty} g(x) \to -\infty$ OR $\lim\limits_{x\to\infty} g \to -\infty$
  - $\lim\limits_{x\to\infty} g(x) \quad -\infty$ OR $\lim\limits_{x\to\infty} g \quad -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim\limits_{x\to-\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing "$x \to \infty$"
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

C **(i)** Based on the table, which of the following function types best models function *f*: linear, quadratic, exponential, or logarithmic?

(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of *f* and the change in the input values of *f*. Refer to the values in the table in your reasoning.

(i) An exponential function best models *f*.	Answer	**Point C1**
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	**Point C2**

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of "exponential" earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR "successive output values are decreasing proportionately by a factor of 0.5." The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references "exponential regression," "*r* values," OR "r^2 values" is not sufficient to earn **Point C2**.
- Special case: A response that indicates that *f* is best modeled by a **linear**, **quadratic**, or **logarithmic** function in part C (i) without a reason in part C (i) <u>combined</u> with a response in part C (ii) that provides both the correct answer <u>and</u> a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.