2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.
Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

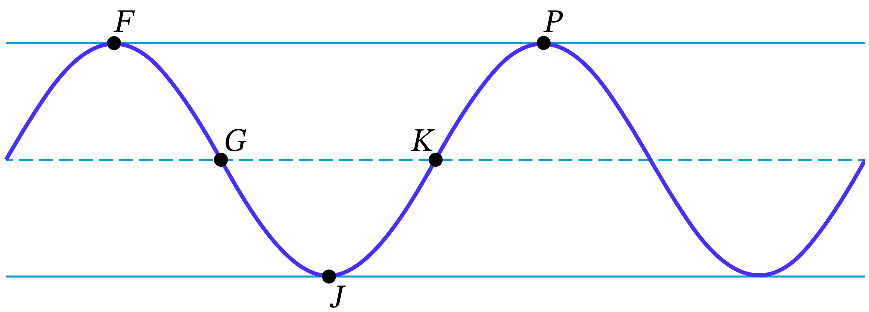
END OF PART A

3. For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented. Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Question 2: Modeling a Non-Periodic Context
Part A: Graphing calculator required — 6 points

A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times *t* months after the musician began using the app.
At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand.
At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function *D* given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and *t* is the number of months after the musician began using the app.

	Model Solution	Scoring	
A	**(i)** Use the given data to write three equations that can be used to find the values for constants *a*, *b*, and *c* in the expression for $D(t)$. **(ii)** Find the values for *a*, *b*, and *c* as decimal approximations.		
	(i) Because $D(0) = 25$, $D(2) = 30$, and $D(4) = 34$, three equations to find *a*, *b*, and *c* are $a(0)^2 + b(0) + c = 25$ $a(2)^2 + b(2) + c = 30$ $a(4)^2 + b(4) + c = 34$	Three equations	**Point A1**
	(ii) $c = 25$ $\begin{array}{r} 4a + 2b = 5 \\ \underline{16a + 4b = 9} \end{array} \Rightarrow \begin{array}{r} 16a + 8b = 20 \\ \underline{16a + 4b = 9} \end{array} \Rightarrow 4b = 11$ $b = \frac{11}{4} = 2.75$ $a = -0.125$ $D(t) = -0.125t^2 + 2.75t + 25$	Values of *a*, *b*, and *c*	**Point A2**

General Scoring Notes for Question 2 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.
- Parts of Question 2 require decimal answers. The first response in Question 2 that is complete, correct, and uses exact values rather than decimal form does not earn the point. For each additional part of Question 2 that requires a decimal answer, a response that is complete, correct, and uses exact values rather than decimal form is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for presenting three equations involving a, b, and c that use the given input-output pairs.
- **Point A2** is earned for correct values of a, b, and c **with or without** supporting work. If correct values are identified, work should be ignored.
- **Point A2** is earned for correct values of a, b, and c presented as either standalone values OR in an expression for $D(t)$.
- A response is eligible to earn both **Point A1** and **Point A2** with a correct translation to "thousands." Use of 25,000, 30,000, and 34,000 results in values of $a = -125$, $b = 2750$, and $c = 25{,}000$.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has two correct equations in the presence of three equations involving a, b, and c AND one correct value.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

B **(i)** Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

(i) $\dfrac{D(4) - D(0)}{4 - 0} = \dfrac{(34 - 25)}{4} = 2.25$ The average rate of change is 2.25 thousand plays per month.	Average rate of change	**Point B1**

(ii) Estimates using the average rate of change can be given by $y = 25 + 2.25t$. For $t = 1.5$, $y = 25 + 2.25 \cdot 1.5 = 28.375$. The estimate of the total number of plays for the song for $t = 1.5$ months is 28.375 thousand.	Estimate using average rate of change	**Point B2**

Scoring Notes for Part B (i)–(ii)

- **Point B1** and **Point B2** both require supporting work.
- **Point B1** is earned for a correct decimal approximation in the presence of a quotient with a difference that uses the given data values. In part B (i) units are not needed and are ignored if presented.
- Another form of the secant line, derived from $D(4) = 34$, is $y = 34 + 2.25(t - 4)$.
- Eligibility for **Point B2**:
  - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
    - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).
- The final number in part B (ii) may be reported as 28 thousand provided the supporting work has a correct decimal approximation for the estimate.
- If a response has a correct translation to "thousands":
  - The response is eligible to earn both **Point B1** and **Point B2**.
  - Use of 25,000 and 34,000 results in an answer of 2250 in part B (i).
  - If a response earned **Point B1** without a decimal presentation error, then an estimate of 28,375 earns **Point B2** in the presence of supporting work.
  - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
    - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).

Partial Credit for Part B (i)–(ii)

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Correct quotient in part B (i) that uses the given data values that is not expressed as a decimal approximation	Correct estimate of 28.375 in part B (ii) that does not include supporting work
Correct quotient in part B (i) that uses the given data values and has a decimal presentation error	Correct or consistent supporting work in part B (ii) that does not provide an estimate
Correct average rate of change in part B (i) that does not include supporting work	

B **(iii)** Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.

Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

(iii) The estimate A_t is the y-coordinate of a point on the secant line that passes through $(0, D(0))$ and $(4, D(4))$. Because the graph of D is concave down on the interval $0 < t < 4$, this secant line is below the graph of D on the interval $0 < t < 4$. Therefore, the estimate using the average rate of change A_t is less than the value of $D(t)$ for all t on the interval $0 < t < 4$.	Explanation	**Point B3**

Scoring Notes for Part B (iii)

To earn **Point B3**, the explanation must include:

- The graph of D is concave down OR the rate of change of D is decreasing.
- A reference to the use of a secant line on $0 < t < 4$ OR the use of a linear function with reference to t-values 0 and 4 that provide the placement of the line.

C	The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D. Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.		
	The total number of plays for the song must be nonnegative and never decreasing. The quadratic function D has an absolute maximum at $t = 11$. The function D increases before $t = 11$ and decreases after $t = 11$. Because D is also positive for positive t-values less than $t = 11$, the domain of model D is an interval with right endpoint $t = 11$.	Explanation	**Point C1**

Scoring Notes for Part C

- A response that focuses on the left endpoint of the domain is eligible for **Point C1**.
- To earn **Point C1**, the explanation must include one of the following:
  - The total number of plays cannot decrease AND the function model D decreases after the time it achieves its absolute maximum.
    - The location of the absolute maximum at $t = 11$ is not required but must be correct if included.
  - The total number of plays is nonnegative AND the left endpoint of the domain is the only value of t at which $D(t)$ changes from negative to positive.
    - $D(t)$ changes from negative to positive at $t = -6.916$. This value is not required, but if included it must be accurate to at least one place after the decimal point, rounded or truncated.