4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions *j* and *k* are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x\left(\csc^2 x - 1\right)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of *m* that yield an output value of 0.

STOP

END OF EXAM

Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed — 6 points

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

	Model Solution	Scoring	
A	The functions g and h are given by $g(x) = 2\log_3 x$ $h(x) = 4\cos^2 x$ **(i)** Solve $g(x) = 4$ for values of x in the domain of g. **(ii)** Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.		
	(i) $g(x) = 4$ $2\log_3 x = 4$ $\log_3 x = 2$ $3^2 = x$ $x = 9$	Solution to $g(x) = 4$	**Point A1**
	(ii) $h(x) = 3$ $4\cos^2 x = 3$ $\cos^2 x = \frac{3}{4}$ $\cos x = \pm\frac{\sqrt{3}}{2}$ Because x is in $\left[0, \frac{\pi}{2}\right)$, $x = \frac{\pi}{6}$	Solution to $h(x) = 3$	**Point A2**

Scoring Notes for Part A

- **Point A1** and **Point A2** both require supporting work. "Scratchwork" can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a complete and correct solution may be considered scratchwork. The use of "$x =$" is not required.
- A logarithmic expression that adds one or both parentheses around the full argument of the logarithm is eligible to earn **Point A1**.
- A response that includes correct values of x outside of the interval $\left[0, \frac{\pi}{2}\right)$ is eligible to earn **Point A2** (e.g., $x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$, or $x = \frac{11\pi}{6}$).
- The use of $\pm$ is not required in supporting work for **Point A2**.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point. Rationalizing denominators is not required.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

First Column	**Second Column**
Correct answer in part A (i) without supporting work.	Correct answer in part A (ii) without supporting work.
Correct answer in part A (i) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions (e.g., $x = 3^2$). No incorrect work.	Correct answer in part A (ii) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions. This includes an answer of $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. No incorrect work.
Answer in part A (i) is reported as $x^2 = 3^4$ OR $x^2 = 81$. No incorrect work follows.	Answer in part A (ii) is reported as $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\cos x = \frac{\sqrt{3}}{2}$. No incorrect work follows.

B The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$

$$k(x) = \frac{6}{\tan x\left(\csc^2 x - 1\right)}$$

(i) Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

(ii) Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

(i) $j(x) = \log_2 x + 3\log_2 2$ $j(x) = \log_2 x + \log_2 2^3$ $j(x) = \log_2(8x),\ x > 0$	Expression for $j(x)$	**Point B1**
(ii) $k(x) = \dfrac{6}{\tan x\left(\csc^2 x - 1\right)}$ $k(x) = \dfrac{6}{\tan x\left(\cot^2 x\right)}$ $k(x) = \dfrac{6}{\cot x}$ $k(x) = 6\tan x,\ \tan x \neq 0,\ \cot x \neq 0$	Expression for $k(x)$	**Point B2**

Scoring Notes for Part B

- **Point B1** is earned with a correct expression for $j(x)$ without supporting work, provided no incorrect work is included. "Scratchwork" can be ignored; the use of a variable other than x is acceptable. The use of "$j(x) =$" is not required.
- **Point B2** requires supporting work. Scratchwork can be ignored; the use of a variable other than x is acceptable. The use of "$k(x) =$" is not required.
- Domain restrictions are not required to be included and are not scored regardless if correct or incorrect.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn **Point B1**.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	**Second Column**
Expression in part B (i) is reported as $\log_2 x + 3$. No incorrect work follows.	Correct expression in part B (ii) without supporting work.
Expression in part B (i) is reported as $\log_2\left(2^3 \cdot x\right)$. No incorrect work follows.	Expression in part B (ii) is reported as $\frac{6}{\cot x}$ OR $\frac{6\sin x}{\cos x}$. No incorrect work follows.
Expression in part B (i) is reported using logarithm base b, $b > 0$, and $b \neq 2$, and has the correct argument.	Expression in part B (ii) includes a correct application of a Pythagorean identity with no incorrect work.

C The function m is given by

$$m(x) = e^{2x} - e^{x} - 12.$$

Find all input values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow\ e^{2x} - e^{x} - 12 = 0$ $\left(e^{x}\right)^2 - e^{x} - 12 = 0$ Let $y = e^{x}$.	Quadratic form with e^{x}	**Point C1**
$y^2 - y - 12 = 0$ $(y-4)(y+3) = 0$ $y - 4 = 0$ or $y + 3 = 0$ $e^{x} = 4$ or $e^{x} = -3 \Rightarrow x = \ln 4$	Value of x	**Point C2**

Scoring Notes for Part C

- **Point C1** and **Point C2** both require supporting work. “Scratchwork” can be ignored; the use of a variable other than x is acceptable. The use of “$x =$” is not required.
- **Point C1** is earned for a substitution of $y = e^{x}$ and factored form of $(y \pm 4)(y \pm 3)$ [the use of a variable other than y is acceptable] OR for presenting $m(x)$ in factored form as $\left(e^{x} \pm 4\right)\left(e^{x} \pm 3\right)$.
- To earn **Point C2**, no incorrect values for x are included.