3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

(a) One person from those surveyed will be selected at random.

(i) What is the probability that the person selected will be someone whose response is never and who is a woman?

(ii) What is the probability that the person selected will be someone whose response is never or who is a woman?

(iii) What is the probability that the person selected will be someone whose response is never given that the person is a woman?

(b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.

(c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

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4. Tumbleweed, commonly found in the western United States, is the dried structure of certain plants that are blown by the wind. Kochia, a type of plant that turns into tumbleweed at the end of the summer, is a problem for farmers because it takes nutrients away from soil that would otherwise go to more beneficial plants. Scientists are concerned that kochia plants are becoming resistant to the most commonly used herbicide, glyphosate. In 2014, 19.7 percent of 61 randomly selected kochia plants were resistant to glyphosate. In 2017, 38.5 percent of 52 randomly selected kochia plants were resistant to glyphosate. Do the data provide convincing statistical evidence, at the level of $\alpha = 0.05$, that there has been an increase in the proportion of all kochia plants that are resistant to glyphosate?

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Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use information from a two-way table of relative frequencies to compute joint, marginal, and conditional probabilities; (2) recognize whether two events are independent; and (3) compute a probability for a binomial distribution.

Solution

Part (a):

(i) $P(\text{never and woman}) = 0.0636$

(ii)

$$\begin{aligned} P(\text{never or woman}) &= P(\text{never}) + P(\text{woman}) - P(\text{never and woman}) \\ &= 0.12 + 0.53 - 0.0636 \\ &= 0.5864 \end{aligned}$$

(iii) $P(\text{never} \mid \text{woman}) = \dfrac{P(\text{never and woman})}{P(\text{woman})} = \dfrac{0.0636}{0.53} = 0.12$

Part (b):

Yes, the event of being a person who responds never is independent of the event of being a woman because

$P(\text{never} \mid \text{woman}) = P(\text{never}) = 0.12.$

Part (c):

Define X as the number of people in a random sample of five people who always take their medicine as prescribed. Then X has a binomial distribution with $n = 5$ and $p = 0.54,$ and

$$\mathrm{P}(X \geq 4) = \binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0 \approx 0.19557 + 0.04592 \approx 0.24149.$$

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response reports correct values of the probabilities for (i), (ii), and (iii).

Partially correct (P) if only one or two of the probabilities are correct.

Incorrect (I) if none of the probabilities are correct.

Notes:

- Assuming independence for events never and woman in (i) without referencing the result in part (b) does not satisfy (i).
- Alternative solutions for (ii) include $0.0564 + 0.0636 + 0.1384 + 0.3280 = 0.5864$ and $0.0564 + 0.53 = 0.5864$.

Question 3 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response indicates that the events are independent, gives an explanation of independence using the events in the problem, *AND* provides appropriate justification using numbers from the table.

Note: Examples of valid explanations with appropriate justifications include:

- $P(\text{woman and never}) = 0.0636$ is the same as $P(\text{woman}) \times P(\text{never}) = (0.53)(0.12) = 0.0636$.
- $P(\text{never} \mid \text{woman}) = \dfrac{0.0636}{0.53} = 0.12$ is the same as $P(\text{never}) = 0.12$.
- $P(\text{woman} \mid \text{never}) = \dfrac{0.0636}{0.12} = 0.53$ is the same as $P(\text{woman}) = 0.53$.

Partially correct (P) if the response indicates that the events are independent *AND* gives an explanation of independence using the events in the problem but does not provide justification using numbers from the table

OR

if the response uses a correct method of illustrating that events are independent but makes an arithmetic mistake or a transcription mistake that results in concluding that these two events are not independent.

Incorrect (I) if the response does not satisfy requirements for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Clearly indicates a binomial distribution with $n = 5$ and $p = 0.54$.
2. Indicates the correct boundary value and direction of the event.
3. Reports the correct probability.

Partially correct (P) if the response satisfies component 1 but it does not satisfy one or both of the other two components

OR

if the response does not satisfy component 1 but both of the other two components are satisfied.

Incorrect (I) if the response does not meet the criteria for E or P.

Question 3 (continued)

Notes:

- The response $B(5, 0.54)$ satisfies component 1.
- Components 1 and 2 are satisfied by displaying the correct formula for computing the binomial probability using the correct values for n and p, e.g.,

  $$\binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0$$
- Only component 1 is satisfied if the correct binomial distribution is used in an incorrect probability formula, e.g.,

  $$\binom{5}{4}(0.54)^4(0.46)^1$$
- For component 2, the boundary value and direction may be described in words, e.g., *P*(at least four people).
- Component 2 may be satisfied by displaying a bar graph of a binomial distribution with the appropriate bars shaded.
- The response of $1 - \text{binomcdf}\left(n = 5,\ p = 0.54,\ \text{upper bound} = 3\right) = 0.24$ is scored E since n, p, and the boundary value are clearly identified.
  The response of $1 - \text{binomcdf}\left(n = 5, p = 0.54, 3\right) = 0.24$ is scored P since n and p are clearly identified and the boundary value is not identified.
  The response of $1 - \text{binomcdf}\left(5, 0.54, 3\right) = 0.24$ is scored I.
- A normal approximation to the binomial is not appropriate since $np = 5 \times 0.54 = 2.7$ and $2.7 < 5$. A response using the normal approximation can score at most P. To score P, the response must include all of the following:
  - an indication that the probability calculated is a normal approximation for the binomial probability
  - a correct mean and standard deviation based on the binomial parameters
  - clear indication of boundary and direction with a *z*-score or diagram
  - the probability computed correctly.

  An example of a response which meets these four criteria is
  $P\left(Z \geq \dfrac{4 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \geq \dfrac{4 - (5)(0.54)}{\sqrt{(5)(0.54)(0.46)}}\right) \approx 0.1217,$ and the binomial distribution is mentioned.