Begin your response to **QUESTION 3** on this page.

3. To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week.

(a) Consider the probability that a particular employee receives at least one gift card in a 52-week year.

(i) Define the random variable of interest and state how the random variable is distributed.

(ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.

GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Continue your response to **QUESTION 3** on this page.

(b) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.

(c) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 4** on this page.

4. The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered \$10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 customers who placed an order in the past year are selected at random. Each of the selected customers is sent an e-mail with a coupon for \$10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order.

   (a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer.

GO ON TO THE NEXT PAGE.

Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) **(i)** Let the random variable of interest X represent the number of gift cards that a particular employee receives in a 52-week year. Because each employee has probability $\frac{1}{200} = 0.005$ of being selected each week to receive a gift card and each week's selection is independent from every other week, X has a binomial distribution with $n = 52$ repeated independent trials and probability of success $p = 0.005$ for each trial. **(ii)** The probability that a particular employee receives at least one gift card in a 52-week year is: $P(X \geq 1) = 1 - P(X = 0)$ $= 1 - \binom{52}{0}(0.005)^0(0.995)^{52}$ $= 1 - 0.7705$ $= 0.2295$	**Essentially correct (E)** if the response satisfies the following four components: 1. Defines the random variable as the number of gift cards that a particular employee receives in a 52-week year 2. Describes the distribution as binomial with $n = 52$ and $p = 0.005$ 3. Identifies the event of interest (i.e., identify the correct boundary AND direction for the event) in the calculation of the probability in part (a-ii) 4. Provides supporting work to identify the correct probability of 0.2295 (or 0.230, if rounded) OR a probability consistent with components 2 and 3 **Partially correct (P)** if the response satisfies only two or three of the four components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that states $X \sim B(52, 0.005)$ satisfies component 2.
- A response that states the random variable is distributed by a distribution that is not binomial (e.g., normal or uniform) and then uses the binomial calculation does not satisfy component 2.
- Stating that gift cards are distributed randomly is not a distribution and does not, in itself, satisfy component 2. Component 2 can still be satisfied if the response goes on to use the binomial distribution.
- In order to satisfy component 2 using calculator function notation, the sample size and probability parameter must be clearly identified.
  - The following satisfy component 2:
    - binomcdf(n or trials = 52, $p = 0.005$, 1, 52)

    - $1 -$ binomcdf(n or trials = 52, $p = 0.005$, 0)
    - $1 -$ binompdf(n or trials = 52, $p = 0.005$, 0)
  - The following do not satisfy component 2 because the parameter or sample size is not clearly labeled:
    - binomcdf(52, 0.005, lower bound = 1, upper bound = 52)
    - $1 -$ binomcdf(52, $p = 0.005$, upper bound or $x = 0$)
    - $1 -$ binompdf(n or trials = 52, 0.005, $x = 0$)
- In order to satisfy component 3, the supporting work must identify the event of interest, i.e., $X \geq 1$, the boundary is 1, and the direction is greater than or equal to, or at least.
  - Possible ways to do this include:
    - Probability notation, e.g. $P(X \geq 1)$, $1 - P(X = 0)$
    - Summing probabilities, e.g. $\sum_{k=1}^{52} \binom{52}{k} (0.005)^k (0.995)^{52-k}$
    - Description in words $P(\text{employee receives at least one gift card})$, $1 - P(\text{employee receives no gift cards})$
    - Graphical, a bar graph of binomial probabilities with appropriate bars shaded
    - Using calculator function syntax with clearly labeled parameters (e.g. $p = 0.005$, $n = 52$) and clearly labeled event boundaries (e.g., lower bound = 1, upper bound = 52)
  - The following satisfy component 3:
    - binomcdf(n or trials = 52, $p = 0.005$, lower bound=1, upper bound = 52)
    - $1 -$ binomcdf(n or trials = 52, $p = 0.005$, upper bound or $x = 0$)
    - $1 -$ binompdf(n or trials = 52, $p = 0.005$, $x = 0$)
  - The following do not satisfy component 3 because the event boundaries are not clearly labeled.
    - binomcdf(n or trials = 52, $p = 0.005$, 1, 52)
    - $1 -$ binomcdf(n or trials = 52, $p = 0.005$, 0)
    - $1 -$ binompdf(n or trials = 52, $p = 0.005$, 0)
- A response will satisfy component 3 if the probability is computed for a geometric distribution with the first success within the first 52 weeks ($X \leq 52$) (e.g. response that states geometcdf(0.005, x or upper bound = 52)).
- Because $np = (52)(0.005) = 0.26$ is less than 5, the normal approximation to the binomial distribution is not an appropriate method to calculate the probability, and a response that uses this method does not satisfy component 4. However, a response that uses the normal approximation to the binomial distribution may satisfy component 3 if it displays the correct mean and standard deviation of the binomial distribution AND provides a clear indication of the appropriate collection of possible outcomes included in the event using a diagram or a ***z***-score, e.g., $1 - P\left(Z \leq \frac{0 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$, $P\left(Z \geq \frac{1 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$, or $P\left(Z \geq \frac{0.5 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$.

Model Solution	Scoring
(b) The expected value for the number of gift cards a particular employee will receive in a 52-week year is $np = 52(0.005) = 0.26.$ If the random process of selecting one employee each week to receive a gift card is repeated for a very large number of years, each employee can expect to receive about 0.26 gift cards per year, on average, or about one gift card every four years.	**Essentially correct (E)** if the response satisfies the following two components: 1. Correctly calculates the expected value AND provides supporting work for the calculation of the expected value 2. Provides a reasonable interpretation of the expected value that includes *at least two* of the following three aspects: • The concept of repeating the selection process over a long period of time • The concept of an average or mean • The context of receiving gift cards **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response may satisfy component 1 if the reported expected value is consistent with the distribution of the random variable identified in the response to part (a-i), AND supporting work for the calculation of the expected value in part (b) is shown.
- Examples of supporting work that satisfies component 1 include:
  - $np = 52(0.005) = 0.26$ or $np = 52(0.005)$
  - $np = \frac{52}{200}$
  - $52(0.005) = 0.26$
  - $np = 0.26,$ if the values of n and p are reported in the response to part (a)
- A response that incorrectly calculates the expected value may still satisfy component 2 using the incorrect expected value in the interpretation.

	Model Solution	Scoring
(c)	No, Agatha's experience does not constitute strong evidence that the selection process was not truly random. In fact, it is quite likely $(\text{probability} = (0.995)^{52} \approx 0.7705)$ that a particular employee will fail to receive a gift card for an entire 52-week year.	**Essentially correct (E)** if the response satisfies the following three components: 1. Indicates that Agatha does not have a strong argument that the selection process was not truly random 2. Provides a relevant probability or expected value 3. Provides an explanation that correctly links the probability or expected value to the decision **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Examples that satisfy component 2:
  - The probability that Agatha will receive at least one gift card in a 52-week year is 0.2295, or the value computed in part (a-ii).
  - The probability that Agatha will fail to receive a gift card for an entire 52-week year is 0.7705, or the complement of the value computed in part (a-ii).
  - The expected value computed in part (b).
  - Stating AT MOST 52 out of 200 employees will win a gift card (or AT LEAST 148 will not win).
- A response that indicates that Agatha does have a strong argument that the selection process was not truly random (or responds "yes") that is adequately supported by an explanation based on an incorrectly calculated probability in part (a-ii) OR an incorrectly calculated expected value in part (b) is scored E.
- If a response gives two arguments, treat them as parallel solutions and score the weaker solution.

Scoring for Question 3	Score
Complete Response Three parts essentially correct	**4**
Substantial Response Two parts essentially correct and one part partially correct	**3**
Developing Response Two parts essentially correct and no part partially correct *OR* One part essentially correct and one or two parts partially correct *OR* Three parts partially correct	**2**
Minimal Response One part essentially correct and no part partially correct *OR* No part essentially correct and two parts partially correct	**1**