Begin your response to **QUESTION 2** on this page.

2. A dermatologist will conduct an experiment to investigate the effectiveness of a new drug to treat acne. The dermatologist has recruited 36 pairs of identical twins. Each person in the experiment has acne and each person in the experiment will receive either the new drug or a placebo. After each person in the experiment uses either the new drug or the placebo for 2 weeks, the dermatologist will evaluate the improvement in acne severity for each person on a scale from 0 (no improvement) to 100 (complete cure).

   (a) Identify the treatments, experimental units, and response variable of the experiment.

   - Treatments:
   - Experimental units:
   - Response variable:

   Each twin in the experiment has a severity of acne similar to that of the other twin. However, the severity of acne differs from one twin pair to another.

   (b) For the dermatologist's experiment, describe a statistical advantage of using a matched-pairs design where twins are paired rather than using a completely randomized design.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) For the dermatologist's experiment, describe how the treatments can be randomly assigned to people using a matched-pairs design in which twins are paired.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 3** on this page.

3. A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable A represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

(a) A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

(b) The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

(i) Define the random variable of interest for the warehouse manager and state how the random variable is distributed.

(ii) Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

GO ON TO THE NEXT PAGE.

Question 2: Focus on Collecting Data — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) Treatments: New drug, placebo. Experimental units: The 72 people who receive the new drug or placebo. Response variable: Improvement in acne severity	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies the treatments as new drug and placebo 2. Identifies the experimental units as the 72 people (subjects, participants, twins) in the experiment 3. Identifies the response variable as the improvement in acne severity **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- To satisfy component 1, identification of the treatments must include both the placebo and the new drug.
- To satisfy component 2, the response must indicate that the experimental units are individual people. The response could refer to participants, subjects, twins, or members of the pairs of twins without explicitly mentioning the number 72. However, a response that states or implies that there are 36 experimental units (e.g., "the pairs of twins") does not satisfy component 2.
- To satisfy component 3, the response must include the context of "acne" and "improvement" (e.g., "improvement in acne severity," "acne improvement score"), but it does not need to include a reference to the scale, the dermatologist, two-week time periods, or treatments. Reasonable synonyms for improvement can be used, such as using "reduction" or "change" or by including the verbal descriptions of the scale ("no improvement" to "complete cure"). However, a description of a binary outcome (e.g., "whether or not the acne improves") does not satisfy component 3.
- For responses that indicate the 36 pairs of twins are the experimental units, component 3 may be satisfied by indicating that the response variable is the improvement in acne severity or by indicating that the response variable is the difference in improvement in acne severity.
- If the response provides parallel solutions (i.e., two or more complete solutions without choosing or indicating which is to be scored), the response is scored based on the weaker of the two solutions. For example, if a response says that the experimental units are "the 72 participants and the scores from 0 to 100," component 2 is not satisfied.

	Model Solution	Scoring
(b)	Improvement scores will vary due to many factors, including initial acne severity, what treatment is received, and other variables such as diet and genetics. Because the pairs of twins are similar in initial acne severity, pairing allows for the variation in improvement scores due to the treatment received to be distinguished from variation due to initial acne severity, unlike in a completely randomized design. Consequently, using the matched-pairs design will provide a more precise estimate of the mean difference in improvement in acne severity for the new drug compared to the placebo and make it easier to find convincing evidence that the new drug is better, if it really is better.	**Essentially correct (E)** if the response describes a statistical advantage of a matched-pairs design AND satisfies the following three components: 1. The advantage pertains to an inference made after collecting the data (e.g., the ability to distinguish between the effects of the treatments or the precision of the estimate of the drug effect) 2. Indicates that the matched-pairs design is better by using a comparative word (e.g., easier, clearer, greater) or by making an explicit comparison to a completely randomized design 3. Includes context (e.g., "drug," "improvement," "acne," or "twins") **Partially correct (P)** if the response describes a statistical advantage of a matched-pairs design AND satisfies one or two of the three components. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- To be considered an advantage of a matched-pairs design, the advantage described must be true for a matched-pairs design and not be true for a completely randomized design. For example, saying that "random assignment allows us to conclude cause-and-effect" is true of both designs. Similarly, "this allows the dermatologist to make conclusions about people with differing acne severity" is true of both designs. Also, "reduces bias" and "reduces variability in the estimates of the individual treatment means" is true of neither design.
- Responses that describe only the set-up of a matched-pairs experiment do not satisfy the requirement to describe an advantage of a matched-pairs design. For example, the response "in a matched-pairs design, the members of each pair will be similar in terms of acne severity" does not describe an advantage. However, "in a matched-pairs design, we can compare two people with similar acne severity" does describe an advantage.
- Advantages of a matched-pairs design that satisfy component 1 include "makes it easier to determine if the drug is effective," "gives a better estimate of the effect of the new drug," "reduces variability in the estimate of the drug effect," "makes the difference between the drug and the placebo more easily distinguishable," and "gives a clearer picture of how well the drug works."
- Advantages of a matched-pairs design that don't satisfy component 1 include "accounts for a source of variability," "controls for potentially confounding variables," "allows you to distinguish variation due to severity from variation due to treatment," "each person can be compared to someone similar," "reduces variability," "more balanced treatment groups," and "more accurate results."

- It is acceptable to provide a disadvantage of a completely randomized design rather than an advantage of the matched-pairs design (e.g., "The completely randomized design will make it harder to find convincing evidence that the new drug is better").
- It is acceptable to use the term "blocking" as a synonym for "pairing."
- A response that states that a matched-pairs design requires a smaller sample size to get power or precision equal to that in a completely randomized design and describes this advantage in context should be scored E.

Model Solution	Scoring
(c) For each pair of twins, label one person as twin A and label the other person as twin B. For each pair of twins, toss a coin. If the coin lands on heads, twin A gets the placebo and twin B gets the active drug. If the coin lands on tails, twin A gets the active drug and twin B gets the placebo. *OR* Label the members of each pair of twins as "Twin 1" and "Twin 2." Using a random number generator, generate an integer from 1 to 2. Give the drug to the twin whose number is selected and the placebo to the twin whose number is not selected. Repeat for all pairs of twins. *OR* Label 1 notecard "A" and another notecard "B." For each pair of twins, shuffle the cards and give one card to each twin. The twin who gets "A" receives the drug and the twin who gets "B" receives the placebo.	**Essentially correct (E)** if the response randomly assigns the two treatments within pairs of twins AND satisfies the following three components: 1. Uses a random process (e.g., flipping a coin, using a random number generator, shuffling cards) that gives each twin in a pair a 50% probability of getting the drug and a 50% probability of getting the placebo 2. Describes how to use the random process to assign one specific twin in each pair to the drug and the other twin to the placebo 3. Indicates that the random assignment process will be completed for each pair of twins **Partially correct (P)** if the response randomly assigns the two treatments within pairs of twins AND satisfies only two of the three components for E. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- A response that does not randomly assign both treatments within pairs of twins should be scored incorrect (I). Examples include a response that describes a completely randomized design, describes a crossover design where each person receives both treatments, uses pairs other than twins, does not use random assignment, or indicates that both twins in a pair receive the same treatment.
- For responses that use slips of paper or selecting items from a hat, the slips must be shuffled (or blindly drawn) or the hat mixed or shaken to have a random process and satisfy component 1.
- To satisfy component 2, the response must describe what to do for each possible outcome of the random process and specify which treatment each twin receives. For example, none of the following descriptions satisfy component 2:
  - "Roll a die. If it is 1–3, give the first twin the drug and the second twin the placebo." (Response doesn't describe what to do if the die is 4–6.)
  - "Have one member of each pair flip a coin. If it is heads, that twin gets the drug. If it is tails, that twin gets the placebo." (Response doesn't indicate what treatment the other twin will receive.)
  - "Flip a coin. If it is heads, give one twin the drug and the other twin the placebo. If it is tails, do the reverse." (Response doesn't specify which twin is getting the drug.)
  - "Label one slip of paper "A" and a second slip of paper "B." Mix them in a hat and have each member of the pair choose one slip." (Response doesn't specify if A represents the new drug or the placebo.)

- Ignore any discussion about randomly selecting 36 pairs of twins to obtain subjects for the experiment. Likewise, ignore any discussion about how to perform the analysis for a paired design (e.g., "subtract the improvement scores for each pair of twins").
- It is acceptable to refer to each pair of twins as a block.