Begin your response to **QUESTION 3** on this page.

3. A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable A represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

(a) A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

(b) The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

(i) Define the random variable of interest for the warehouse manager and state how the random variable is distributed.

(ii) Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

To reduce the number of crates rejected by the beauty supply warehouse manager, the manufacturing company is considering adjusting the programming of the filling machine so that the amount of shampoo in each bottle is normally distributed with mean 0.56 liter and standard deviation 0.03 liter.

(c) Would you recommend that the manufacturing company use the original programming of the filling machine or the adjusted programming of the filling machine? Provide a statistical justification for your choice.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 4** on this page.

4. A survey conducted by a national research center asked a random sample of 920 teenagers in the United States how often they use a video streaming service. From the sample, 59% answered that they use a video streaming service every day.

   (a) Construct and interpret a 95% confidence interval for the proportion of all teenagers in the United States who would respond that they use a video streaming service every day.

GO ON TO THE NEXT PAGE.

Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	Random variable A, which represents the amount of shampoo in a randomly selected bottle, follows a normal distribution with mean 0.6 liter and standard deviation 0.04 liter. Then, the probability that a randomly selected bottle is underfilled is $P(A < 0.5) = P\left(Z < \frac{0.5 - 0.6}{0.04} = -2.5\right) \approx 0.0062.$	**Essentially correct (E)** if the response includes the following three components: 1. Indicates the use of a normal (or approximately normal) distribution and identifies the correct parameter values (mean 0.6 and standard deviation 0.04) 2. Specifies the correct event (boundary value and direction), or an event consistent with values reported in component 1 3. Provides the correct probability of 0.0062 or probability consistent with components 1 and 2 **Partially correct (P)** if the response satisfies only two of the three components *OR* if the response fails to satisfy component 1 and 2, but shows the correct z-score formula, z-score value, and correct probability (e.g., $\frac{0.5 - 0.6}{0.04} = -2.5,$ resulting in a probability of 0.0062). **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

Component 1

- A response may satisfy component 1 by any of the following or a combination of the following:
  - Graphical: Displaying a graph of a normal density function with the appropriate scale on the horizontal axis showing the mean and standard deviation for the distribution of shampoo amount.

- Words or standard notation: Specifying the correct event in words with identification of the correct numerical boundary and correct direction, such as "probability that X is at least two" or "probability that at least two bottles are underfilled" or P(at least two bottles are underfilled). Identification of the distribution and parameters may be obtained from the response to part (b-i).
- Random variable: $P(X \geq 2)$ or $1 - P(X \leq 1)$. Identification of the distribution and parameters may be obtained from the response to part (b-i).
- Probability formula: e.g., $1 - \binom{10}{1}(0.0062)^1(0.9938)^9 - \binom{10}{0}(0.0062)^0(0.9938)^{10}$.
- Calculator function notation: Using calculator function notation with clearly defined arguments. For example:
  - "1 – binomcdf$(n = 10,\ p = 0.0062,$ upper bound $= 1)$" satisfies component 3 because the binomial parameters and the boundary value are clearly labeled.
  - "1 – binomcdf$(n = 10,\ p = 0.0062, 1)$" does not satisfy component 3 because the boundary value is not labeled.
  - "1 – binomcdf$(10,\ 0.0062,$ upper bound $= 1)$" does not satisfy component 3 because the binomial parameters are not labeled.

- Because $np = (10)(0.0062) = 0.062$ is less than 5, the normal approximation to the binomial distribution is not an appropriate method to calculate the probability, and a response that uses this method does not satisfy component 3. However, a response that uses the normal approximation to the binomial distribution may satisfy component 4 if it displays the correct mean and standard deviation of the binomial distribution AND provides a clear indication of the appropriate collection of possible outcomes included in the event using a diagram or a z-score, e.g., $P\left(Z \geq \frac{2 - (10)(0.0062)}{\sqrt{(10)(0.0062)(0.9938)}}\right)$ or $1 - P\left(Z \leq \frac{1 - (10)(0.0062)}{\sqrt{(10)(0.0062)(0.9938)}}\right)$. (Note that $\sqrt{(10)(0.0062)(0.9938)} \approx 0.248$.)
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(c) The company should use the original programming for the filling machine. For the original programming of the filling machine, the probability of an underfilled bottle is $P(A < 0.5) = P\left(Z < \frac{0.5 - 0.60}{0.04}\right)$ $= P(Z < -2.5) \approx 0.0062.$ For the adjusted programming of the filling machine, the probability of an underfilled bottle is $P(A < 0.5) = P\left(Z < \frac{0.5 - 0.56}{0.03}\right)$ $= P(Z < -2.0) \approx 0.02275.$ Because the probability of an underfilled bottle is greater for the adjusted programming, this would result in more rejected shipments. The company should continue with the original machine programming.	**Essentially correct (E)** if the response satisfies the following two components by comparing either probabilities or z-scores: Comparing probabilities: 1. Correctly calculates the probability of underfilling a bottle as 0.023 for the adjusted programming of the filling machine 2. Provides a correct conclusion about which programming (adjusted or original) should be recommended based on a comparison of the probabilities calculated for the original and adjusted programming *OR* Comparing z-scores: 1. Correctly calculates the z-score for the adjusted programming 2. Provides a correct conclusion about which programming (adjusted or original) should be recommended based on a comparison of the z-scores (e.g., a higher z-score results in more bottles being underfilled) calculated for the original and adjusted programming **Partially correct (P)** if the response satisfies only one of the two components required for an E. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

- A response that correctly uses the binomial distribution to find the probability that a crate will be rejected with correct values and justification should be scored E. For the original programming, this probability is 0.0017, and for the adjusted programming, this probability is 0.0206.
  Adjusted programming:
  Let Y represent the number of underfilled shampoo bottles in a box of 10 using the adjusted programming.
  $P(Y \geq 2) = 1 - P(Y \leq 1)$
  $= 1 - \binom{10}{1}(0.02275)^1(0.97725)^9$
  $-\binom{10}{0}(0.02275)^0(0.97725)^{10}$
  ≈ 0.0206
- A response that incorrectly computes the probability that a crate will be rejected, with or without justification, should be scored P if it provides a correct conclusion based on comparing that probability to the probability computed in part (b-ii).

- Component 2 is not satisfied if no recommendation is made for choice of programming. A response stating "yes" or "no" is not sufficient for indicating a choice of programming.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.