Begin your response to **QUESTION 3** on this page.

3. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable, X, represent the dollar value of the cash prize in a bath fizzy. The probability distribution of X is shown in the table.

Cash prize, x	\$1	\$5	\$10	\$20	\$50	\$100
Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(a) Based on the probability distribution of X, answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain \$1.

(ii) Calculate the proportion of bath fizzies that contain at least \$10.

(b) Based on the probability distribution of X, calculate the probability that a randomly selected bath fizzy contains \$100, given that it contains at least \$10. Show your work.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(c) Based on the probability distribution of X, calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work.

(d) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 4** on this page.

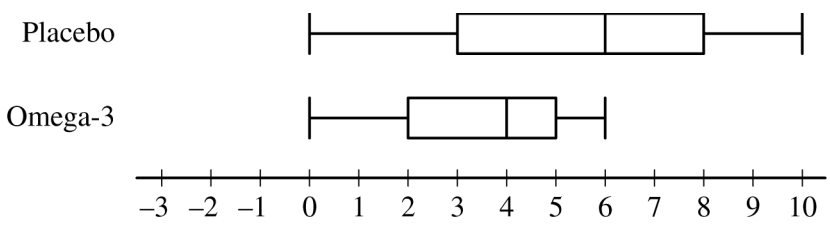
4. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

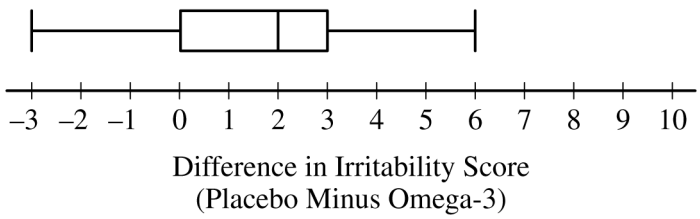
   - Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
   - Week 2: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
   - Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

   At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

   For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots.

	n	Mean	Standard Deviation
Placebo	19	5.421	2.987
Omega-3	19	3.632	1.739
Difference (placebo minus omega-3)	19	1.789	2.485





GO ON TO THE NEXT PAGE.

Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	The random variable X is the dollar value of the cash prize in a bath fizzy. **(i)** The proportion of bath fizzies containing \$1 is equal to the $P(X = \$1)$ and $P(X = \$1)$ $= 1 - (0.2 + 0.05 + 0.05 + 0.01 + 0.01)$ $= 0.68$. **(ii)** The proportion of bath fizzies that contain at least \$10 is equal to the $P(X \geq \$10)$ and $P(X \geq \$10)$ $= 0.05 + 0.05 + 0.01 + 0.01$ $= 0.12$.	**Essentially correct (E)** if the response satisfies at least three of the following four components: 1. Correctly calculates the proportion of bath fizzies containing \$1 2. Provides correct supporting work for the value calculated in component 1 3. Correctly calculates the proportion of bath fizzies containing at least \$10 4. Provides correct supporting work for the value calculated in component 3 **Partially correct (P)** if the response satisfies only two of the four components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that provides a correct percentage, instead of a proportion, may satisfy components 1 and 3, such as 68 percent or 12 percent.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

	Model Solution	Scoring
(b)	Given a bath fizzy contains at least \$10, then the probability that it contains \$100 is $P(X = \$100 \mid X \geq \$10)$ $= \frac{0.01}{0.12}$ $\approx 0.0833.$	**Essentially correct (E)** if the response satisfies the following two components: 1. Correctly calculates the requested probability 2. Shows work consistent with their response to part (a-ii) **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A specific conditional probability statement is not required, but if correctly given should be considered a positive in holistic scoring.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Scoring for Question 3 Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.	**Score**
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**
If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.	

General

- Minor errors in statistical notation may be ignored. However, this is considered poor communication and may be considered if holistic scoring is required.
- If the only error in part (b) is the reversal of the numerator for the *z*-score $(300 - 303)$, the response is scored P.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Alternate Solution:

- A response that calculates the probability that the ***total*** amount of gold on the two randomly selected necklaces is greater than 606 mg $(2 \times 303\,\text{mg})$ can be scored essentially correct if the response demonstrates:
  - The use of a normal (or approximately normal) distribution and the correct mean and standard deviation for the distribution of the total amount of gold applied to two randomly selected necklaces, (e.g., $N(600, 7.071)$, or mean $= 2(300) = 600$ mg and standard deviation $= \sqrt{2\left(5^2\right)} \approx 7.071$ mg, or $z = \dfrac{606 - 600}{7.071} \approx 0.8485$), satisfying component 1.
  - Specifies the correct event, including the correct boundary value of $x_{total} = 2(303) = 606$ or $z \approx 0.8485$, and the correct direction, satisfying component 2.
  - The probability is correctly computed using the mean and the standard deviation of the total amount of gold applied to two randomly selected necklaces. An arithmetic error can be ignored if correct work is shown and the result is between 0 and 0.5.
- While the probability of a difference in part (b-ii) is twice the probability found in part (b-i), it is not necessary for a response to double the probability in part (b-i). However, if the probability in part (b-i) is doubled, that should be considered if holistic scoring is required.

Model Solution	Scoring
(c) **(i)** The sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$ is skewed to the right. Almost all values of the simulated ranges are between 0 mg and about 25 mg and the center of the distribution is about 6 mg. **(ii)** As the value of the population standard deviation increases, the variation (spread) in the distribution of the sample range increases and the mean of the distribution of the sample range also increases.	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. In part (c-i), describes the shape as positively skewed (or skewed to the right) 2. In part (c-i), describes the center of the distribution as about 6 mg 3. In part (c-i), describes the spread of the simulated sample ranges as having most values between 0 mg and 30 mg 4. In part (c-ii), indicates that the mean (or center) of the distribution of the sample range increases as the population standard deviation increases 5. In part (c-ii), indicates that the variation (or spread) of the distribution of the sample range increases as the population standard deviation increases **Partially correct (P)** if the response satisfies three of the five components *OR* the response satisfies only components 4 and 5 *OR* the response only satisfies components 2 and 4 *OR* the response satisfies only components 3 and 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- The centrality component of the response to part (c-i) may be satisfied with any reasonable description of center, such as a comment on the approximate value of mean or median of the distribution or indication that the center is somewhere between 4 and 10 mg.
- The spread component of the response to part (c-i) may be satisfied with a reasonable statement about the inter-quartile range. Any value between 5 and 7, inclusive, is considered reasonable.
- Although the actual range of the sample distribution of the sample range is infinite, a comment that the range is any value between 20 and 30 may be scored as satisfying the spread component of the response to part (c-i).
- The response to part (c-ii) need not comment on the skewness of the distributions. Any comment on the skewness of the distributions should be considered as extraneous in scoring the response.

Model Solution	Scoring
(d) **(i)** No, a sample range of 10 mg is not unusual if the machine is working properly with a standard deviation of 5 mg. Although observing a sample range around 10 mg or greater is much more likely if the population standard deviation is 8 mg or 12 mg than when the population standard deviation is 5 mg, the graph of the sampling distribution of the sample range for samples of size 2 from a normal distribution with $\sigma = 5$ mg indicates that 10 mg is not an unusual value for the range when $\sigma = 5$ mg. There is about a 20% chance that a random sample of two necklaces would yield a range of 10 mg or more when the machine is working properly. **(ii)** No, Cleo's sample mean of 303 mg and range of 10 mg do not indicate that the machine is not working properly. As noted in part (b-i), the probability that the sample mean would be equal to or greater than 303 mg when the machine is working properly is almost 20% so having a sample mean of 303 mg is not unusual. Furthermore, it is less than one standard deviation, $\frac{5}{\sqrt{2}} = 3.5355$ mg, away from 300 mg. As indicated in part (d-i), the probability of a range of 10 mg or greater when the population standard deviation is 5 mg is also about 20%, so not unusual. There is not statistically significant evidence to show the machine is not working properly.	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. In part (d-i), indicates a sample range of 10 mg is not unusual 2. In part (d-i), justifies the response to component 1 by indicating that the graph of the sampling distribution of the sample range when $\sigma = 5$ mg shows that values of 10 mg or greater occur often 3. In part (d-ii), indicates there is not convincing evidence that the machine is not working properly 4. In part (d-ii), justifies the response by indicating that a sample mean of 303 mg would not be unusual when the machine is working properly with one of the following • 303 mg is less than one standard deviation away from 300 mg • The probability that the mean is greater than or equal to 303 mg was shown in (b) to have probability of 0.198, which is not unusual • Some other reasonable justification 5. In part (d-ii), justifies the response by indicating that a sample range of 10 mg is not unusual when the machine is working properly as discussed in part (d-i) **Partially correct (P)** if the response does not meet the requirements for E and satisfies at least three of the five components *OR* the response satisfies components 1 and 2 *OR* the response satisfies component 3 and either component 4 or 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An argument that a sample range of 10 mg or more is more likely to occur (or closer to the center of the sampling distribution of the range) when $\sigma = 8$ mg or $\sigma = 12$ mg is not required for component 2, but by itself does not satisfy component 2. Inclusion of this argument should be considered if holistic scoring is required.