4. A software application (app) lets users enter questions to receive answers in the form of images, texts, or videos. Research indicates that 22 percent of high school students in Country W use the app to help them with their homework at least once per week. Karen is an AP Statistics student in Country W at a high school that has more than 2,000 students. She believes the proportion of all students at her school who use the app to help them with their homework at least once per week is greater than the proportion for her country. To investigate her belief, she took a simple random sample of 130 students from her school and found that 38 of the sampled students use the app to help them with their homework at least once per week.

Is there convincing statistical evidence, at a 0.05 significance level, to support Karen's belief? Justify your answer with the appropriate inference procedure.

5. According to a 2017 national survey in Country B, the mean number of bedrooms in newly built houses was 2.9. Rodney, a researcher, believes the mean number of bedrooms in newly built houses in the country was different in 2024 than it was in 2017. To investigate his belief, he took a large random sample of newly built houses in Country B in 2024 and recorded the number of bedrooms in each house. The distribution of the number of bedrooms for the sampled houses is summarized in the table.

Distribution of the Number of Bedrooms for the Houses Sampled in 2024

Number of Bedrooms	1	2	3	4	5	6
Proportion of Houses	0.12	0.22	0.28	0.22	0.14	0.02

A.

i. A house from the sample will be selected at random. What is the probability that the house had fewer than 3 bedrooms? Show your work.

ii. What is the mean number of bedrooms for the sample of newly built houses in 2024? Show your work.

B. Rodney will use a one-sample *t*-test for a population mean to test his belief.

i. In the context of Rodney's investigation, state the hypotheses for the test.

ii. Explain, in context, what a Type I error would be for Rodney's hypothesis test.

C. A different researcher, Keisha, suggests using a confidence interval to investigate whether the mean number of bedrooms in newly built houses in 2024 in Country B was different from 2.9.

Assume the conditions for inference have been met. Using Rodney's data, Keisha calculated a one-sample 97 percent confidence interval to estimate the population mean as $(3.01, 3.19)$. Based on the confidence interval, what conclusion can be made for Rodney's hypothesis test in part B at $\alpha = 0.03$? Justify your answer.

Question 4: Focus on Inference — 4 points

General Scoring Notes

- This question is scored in three sections. Each section is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test. The second section includes verifying the conditions for the test identified in the first section and calculating the value of the test statistic and the corresponding *p*-value. The third section includes the conclusion for the test identified in the first section. The response is then categorized based on the scores assigned to each section and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
Section 1	An appropriate inference procedure is a one-sample *z*-test for a population proportion. The null hypothesis is $H_0: p = 0.22$, and the alternative hypothesis is $H_a: p > 0.22$, where $p =$ the true proportion of students at Karen's high school that use the application at least once per week.	**Essentially correct (E)** if the response satisfies the following four components: 1. Identifies a one-sample *z*-test for a population proportion by name (e.g., "one-proportion *z*-test" but not merely "one-sample *z*-test") or by formula 2. States the correct equality for the null hypothesis with the value 0.22 3. States the correct direction for the one-sided alternative hypothesis consistent with the null hypothesis 4. Provides sufficient context for the parameter by including reference to the population proportion (true proportion of students at Karen's high school) AND the sampling units (students) AND the response variable (using the application) **Partially correct (P)** if the response satisfies three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- If the response identifies the correct test by name but also states an unreasonable formula, then component 1 is not satisfied.
- If the response identifies the test using the correct formula but equating it with a *t* instead of a *z*, then component 1 is not satisfied.
- A response that states the null hypothesis as $H_0: p \leq 0.22$ satisfies component 2.

- Components 2 and 3 may be satisfied without regard to the symbol (or lack of symbol) used to represent the population parameter.
- Neither context nor the concept of the population is required to satisfy components 2 or 3.
- A response that states the hypotheses in words (e.g., "the null hypothesis is that the proportion is 0.22, and the alternative hypothesis is that the proportion is greater than 0.22") may satisfy components 2 and 3.
- A response that states the hypotheses in words and refers to the population in context (e.g., "the null hypothesis is that the population proportion of students at Karen's high school that use the application at least once per week is equal to 0.22 and the alternative hypothesis is that the population proportion is greater than 0.22") may satisfy components 2, 3, and 4.
- If the null hypothesis is incorrect, the response can satisfy component 3 with a correct alternative hypothesis.
- The elements of component 4 do not have to be satisfied with the statement of the hypotheses. They may be satisfied with the statement in the hypotheses, definition of the parameter, or the statement of the conclusion.
- If the statement of the hypotheses refers to population proportion and the conclusion refers to sample proportion (or vice versa), then the population aspect of component 4 is not satisfied.
- If the response clearly refers to the **sample** proportion instead of the **population** proportion using a $\hat{p}$, then component 4 is not satisfied unless the symbol used is defined as the **population** proportion.
- A response may satisfy the population aspect of component 4 by the following:
  - Referring to the population by using words such as "population," "all," or "true" when defining the parameter or in the statement of the conclusion of the inferential procedure.
  - Using notation such as p, p_0, or π when defining the hypothesis statements.

Confidence Interval Approach:

- If a one-sample z-interval for a population proportion is identified correctly by name (e.g., "one-proportion z-interval" but not merely "one-sample z-interval") or by formula, then component 1 is satisfied.
- If a response uses a one-sample z-interval for a population proportion, then component 4 is satisfied if the response indicates that it is a confidence interval for the true proportion of students at Karen's high school that use the application at least once per week.

	Model Solution	Scoring
Section 2	The independent observation condition for performing the one-sample z-test for a population proportion is satisfied. This is because the data were obtained from a random sample of 130 high school students from Karen's high school. Also, the sample of 130 students is less than 10% of the total number of students at this large high school, because $130 < 0.10(2{,}000)$ and the total number of students in Karen's high school is greater than 2,000. The 10% condition is required as sampling was conducted without replacement from a finite population. The number of expected successes and expected failures were both more than 10 because $130(0.22) = 28.6$ and $130(0.78) = 101.4$. Thus, the sample size is large enough to support the assumption that the sampling distribution of $\hat{p}$ is approximately normal. $\hat{p} = \frac{38}{130} \approx 0.2923$ Test statistic: $z = \frac{0.2923 - 0.22}{\sqrt{\frac{0.22(1-0.22)}{130}}} \approx 1.99$ $P(z > 1.99) \approx 0.023$	**Essentially correct (E)** if the response satisfies the following four components: 1. Checks the independence condition by referring to the random sample of 130 students AND indicates that 130 is less than or equal to 10% of 2,000 (i.e., $(130) \le (0.10)N$) 2. Checks that the sample size is large enough to support the assumption that the sampling distribution of $\hat{p}$ is approximately normal by verifying that the number of expected successes and expected failures were at least 10 by calculating the following values: $np_0 = 130(0.22) = 28.6$ and $n(1 - p_0) = 130(0.78) = 101.4$ 3. Correctly reports the value of the z-statistic 4. Reports the value for the correct p-value consistent with the test statistic and the procedure stated in Section 1 **Partially correct (P)** if the response satisfies only two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- To satisfy the reference to the random selection of 130 students in component 1, it is minimally acceptable to state "random sample – check" or "SRS – check." However, component 1 is not satisfied if the response implies that random **assignment** was used or only states "random – check."
- To satisfy component 2, a direct comparison must be made against a standard criterion (5 or 10) using either actual values of the observed successes and failures (38 and 92) OR values for the expected successes and failures (28.6 and 101.4) OR formulas for the expected number of successes and failures with values inserted (or defined elsewhere), such as $130(0.22)$ and $130(1 - 0.22)$.
- If the response includes an inappropriate check of conditions, such as $n > 30$, then component 2 is not satisfied.
- A response that reports the correct value for the z-statistic but contains errors in supporting work satisfies component 3.
- A response that inputs correct values into the z-statistic formula but computes an incorrect value for the z-statistic, satisfies component 3.

Scoring for Question 4	Score
Complete Response Three sections essentially correct	**4**
Substantial Response Two sections essentially correct and one section partially correct	**3**
Developing Response Two sections essentially correct and no section partially correct *OR* One section essentially correct and one or two sections partially correct *OR* Three sections partially correct	**2**
Minimal Response One section essentially correct and no section partially correct *OR* No section essentially correct and two sections partially correct	**1**