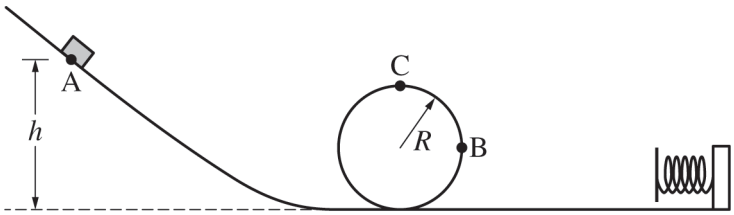
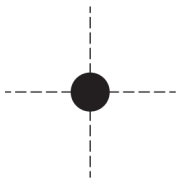
Begin your response to **QUESTION 2** on this page.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

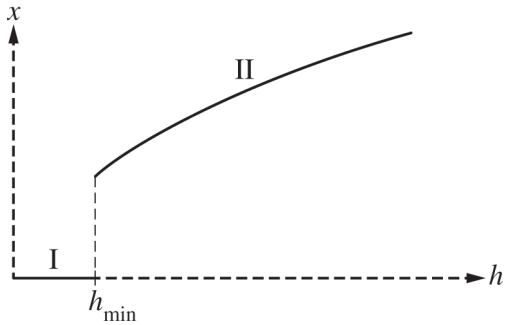
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30\,\text{m}$ and $R = 0.10\,\text{m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

GO ON TO THE NEXT PAGE.

(c)

For including the correct component of weight in a Newton's second law equation for the cart **1 point**

$F_{\text{fan}} + mg\sin\theta = ma_2$

For a correct Newton's second law equation **1 point**

$F_{\text{fan}} + mg\sin\theta = ma_2$

For correct substitutions consistent with part (b) into the above equation **1 point**

$mg\sin\theta = ma_2 - F_{\text{fan}}$

$\theta = \sin^{-1}\left(\dfrac{ma_2 - F_{\text{fan}}}{mg}\right)$

$\theta = \sin^{-1}\left(\dfrac{(0.50\text{ kg})\left(2.4\text{ m/s}^2\right) - (0.50\text{ kg})\left(0.8\text{ m/s}^2\right)}{(0.50\text{ kg})\left(9.8\text{ m/s}^2\right)}\right)$

$\theta = 9.4°$

Total for part (c) 3 points

(d)

For selecting "No" and an attempted justification **1 point**

For a correct justification **1 point**

Example response for part (d)

The mass of the cart cancels out in the equation used to find the angle of the incline.

Total for part (d) 2 points

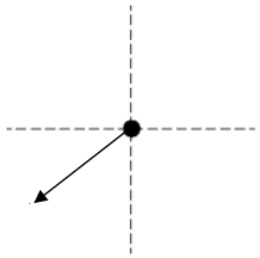
Question 2: Free-Response Question — 15 points

(a) For a single acceleration vector pointing down and to the left and attempting a justification **1 point**

For a correct justification **1 point**

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) 2 points

(b) i. For correctly using conservation of energy for the block at point B **1 point**

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation **1 point**

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h-R)}$$

ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force **1 point**

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B **1 point**

$$F_{net} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{net} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$$

Total for part (b) 4 points

(e) i.	For a correct justification	**1 point**
	Example response for part (e)(i) *Because the block does not make it through the loop at this height, it will not compress the spring.*	
ii.	For indicating that as the height increases, the compression of the spring increases	**1 point**
	For indicating that the height is proportional to the square of the compression of the spring	**1 point**
	Example response for part (e)(ii) *From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.*	
	Total for part (e)	**3 points**
	Total for question 2	**15 points**