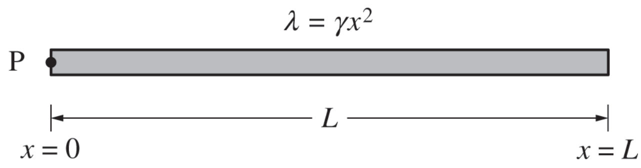
Begin your response to **QUESTION 3** on this page.



3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

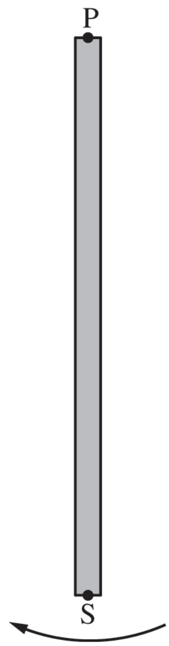
(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing   _____ Decreasing   _____ Not changing

Justify your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.



(f) The mass of the rod is $3.0\,\text{kg}$, and the length of the rod is $1.0\,\text{m}$. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

GO ON TO THE NEXT PAGE.

Question 3: Free-Response Question — 15 points

(a) For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting γx^2 into the above equation — **1 point**

$$I = \int x^2 \left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma\left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

Total for part (a) — 2 points

(b) For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting γx^2 into the numerator of the above equation — **1 point**

For correctly substituting M into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

Total for part (b) — 3 points

(f)	For using conservation of energy to calculate the speed of the rotating rod $U_i + K_i = U_f + K_f$ $U_i + 0 = 0 + K_f$ $U_i = K_f$	**1 point**
	For correctly substituting into the above equation $mgh_i = \frac{1}{2}I\omega_f^2$	**1 point**
	For correctly solving for the linear speed of point S $Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$ $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m})} = 4.9\ \text{m/s}$	**1 point**
	Total for part (f)	**3 points**
	Total for question 3	**15 points**