Begin your response to **QUESTION 3** on this page.

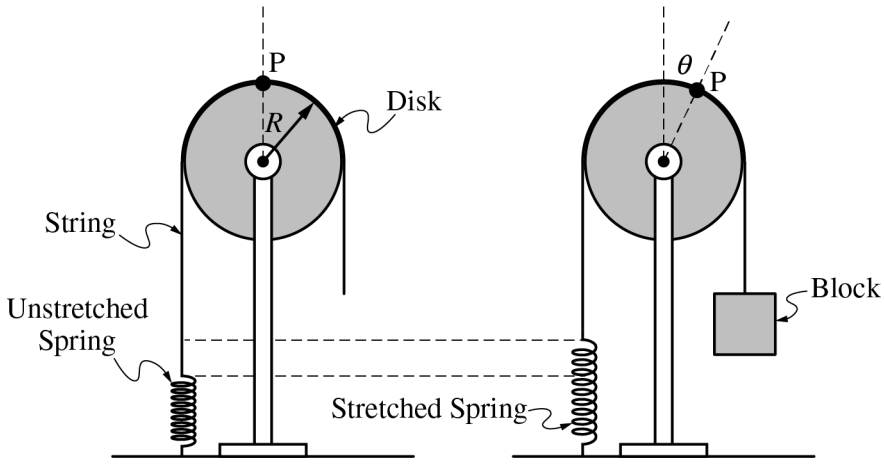

Figure 1 Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2} M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

   A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

   Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

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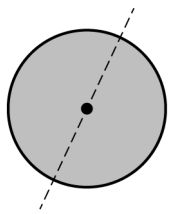
Continue your response to **QUESTION 3** on this page.

(a) Derive an expression for the mass m_B of the block.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.



(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

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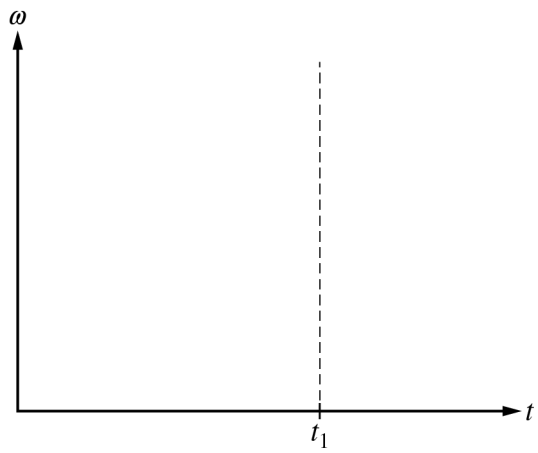
Continue your response to **QUESTION 3** on this page.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.



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Continue your response to **QUESTION 3** on this page.

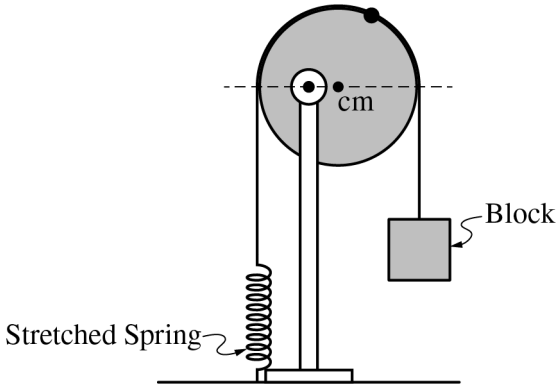

Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

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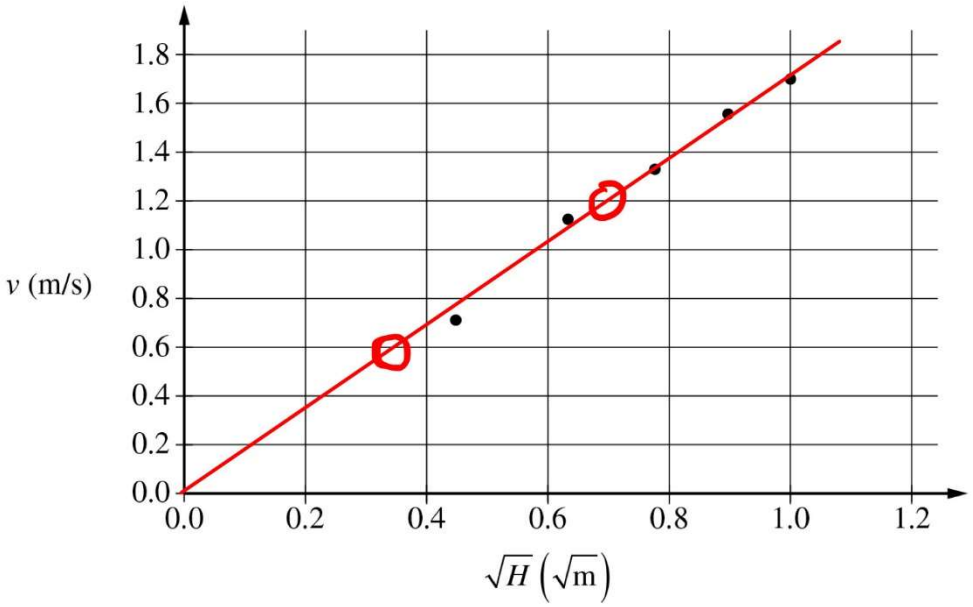
(d)(i) For drawing an appropriate best-fit line including approximately the same number of points above and below the line **1 point**

Example Response



(d)(ii) For calculating the slope using two points on the best-fit line **1 point**

For correctly relating the slope of the best-fit line to the mass of Cart 2 **1 point**

For a correct mass of Cart 2 **1 point**

Example Response

$$slope = \frac{(1.20 - 0.60)\,\text{m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\,\frac{\sqrt{\text{m}}}{\text{s}}$$

$$slope = \frac{m_1}{m_1 + m_2}\sqrt{2g}$$

$$m_2 = \frac{m_1\sqrt{2g}}{slope} - m_1$$

$$m_2 = \frac{(0.25\ \text{kg})\sqrt{2\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)}}{1.72\,\frac{\sqrt{\text{m}}}{\text{s}}} - (0.25\ \text{kg})$$

$$\therefore m_2 = 0.39\ \text{kg}$$

Scoring Note: Acceptable responses for mass are 0.30 to 0.60 kg

Total for part (d) 4 points

Question 3: Free-Response Question — 15 points

(a)

For indicating that the sum of the torques on the disk equals zero — **1 point**

$\Sigma\tau_{\text{on disk}} = 0$

$\tau_{\text{g}} = \tau_{\text{s}}$

OR

For indicating that the sum of the forces equals zero

$\Sigma F = 0$

$F_{\text{g}} = F_{\text{s}}$

For correctly substituting the expressions for the forces — **1 point**

$F_{\text{g}}R = F_{\text{s}}R$

$m_{\text{B}}gR = k\Delta xR$

$m_{\text{B}}g = k\Delta x$

OR

$F_{\text{g}} = F_{\text{s}}$

$m_{B}g = k\Delta x$

For correctly substituting for Δx — **1 point**

$m_{\text{B}}g = kR\theta$

$m_{\text{B}} = \dfrac{kR\theta}{g}$

Total for part (a) — 3 points

(b)

For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk — **1 point**

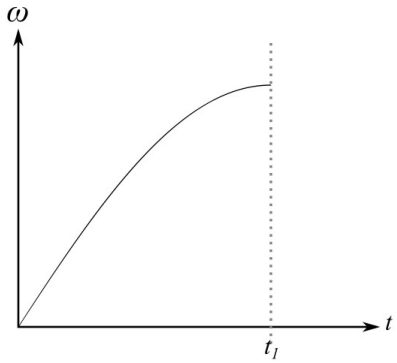
For correct location and label of the force due to gravity exerted on the disk, directed straight down — **1 point**

For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$) — **1 point**

(d) For a sketch that starts at zero and monotonically increases until time $t = t_1$ **1 point**

For a sketch that is concave down between time $t = 0$ and $t = t_1$ **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) 2 points

(e) For indicating that the torque exerted by the force due to gravity on the disk increased **1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased **1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased **1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

Total for part (e) 3 points

Total for question 3 15 points