Begin your response to **QUESTION 2** on this page.

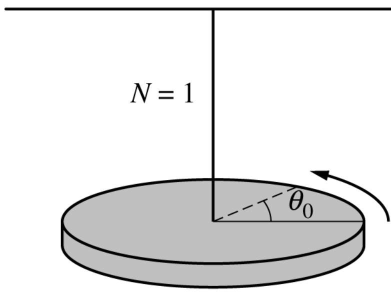

Figure 1

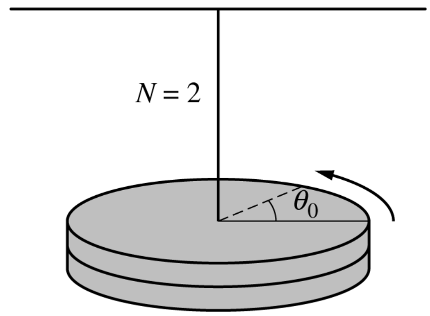

Figure 2

2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

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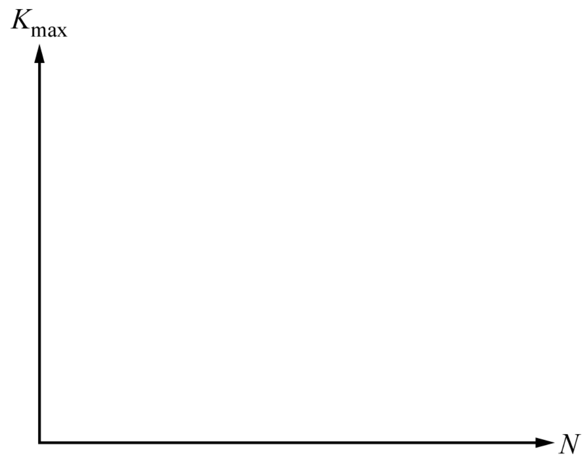
Continue your response to **QUESTION 2** on this page.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.



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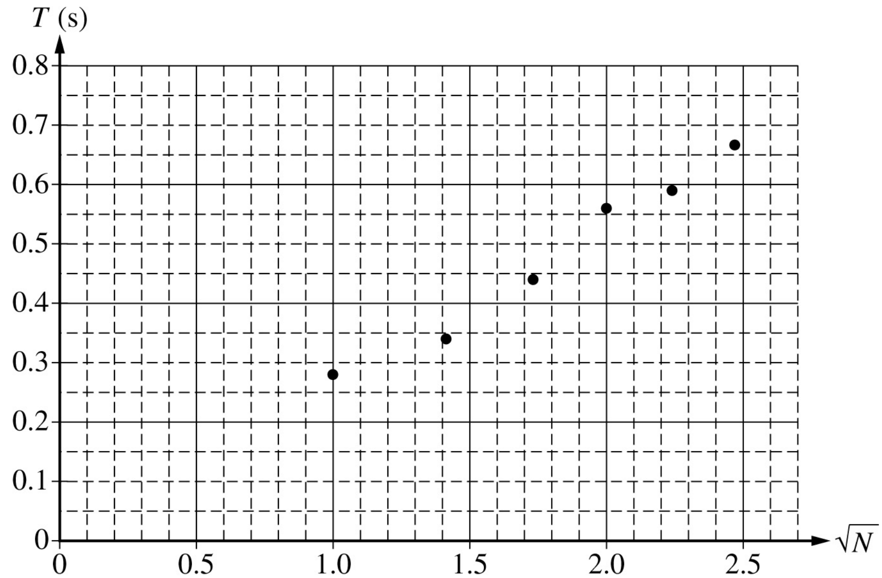
(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



i. Draw the best-fit line for the data.

ii. The student previously determined that the radius of a disk is $R = 0.2\ \text{m}$ and found that $\kappa = 1.6\ \text{N}\cdot\text{m}$. Using the graph, calculate the mass M of a single disk.

iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M. Justify your answer.

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(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than   ___less than   ___the same as

Justify your answer.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_U and $\omega_{\text{non-U}}$?

____$\omega_U > \omega_{\text{non-U}}$ ____$\omega_U < \omega_{\text{non-U}}$ ____ $\omega_U = \omega_{\text{non-U}}$

Briefly justify your answer.

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Question 2: Free-Response Question — 15 points

(a) For indicating the rotational inertia is the sum of the rotational inertia for all the stacked disks — **1 point**

Example Response

$$I_{eq} = \sum_{i=1}^{N} I_i$$
$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$

For an expression for the period consistent with the previous rotational inertia expression — **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{N\left(\frac{1}{2}MR^2\right)}{\kappa}}$$

Example Solution

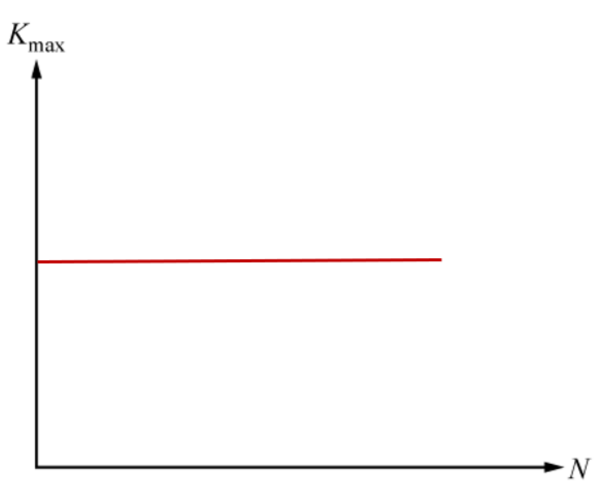
$$I_{eq} = \sum_{i=1}^{N} I_i$$
$$I_{eq} = N\left(\frac{1}{2}MR^2\right)$$
$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$
$$T = 2\pi\sqrt{\frac{N\left(MR^2\right)}{2\kappa}}$$

Total for part (a) — 2 points

(b) For a sketch that begins at a non-zero value — **1 point**

For a sketch that is constant with slope equal to zero — **1 point**

Example Solution



Total for part (b) — 2 points

(d)(ii)	For selecting $\omega_U > \omega_{non\text{-}U}$ with an attempt at a relevant justification, or a selection consistent with part (d)(i)	**1 point**
	For using energy conservation to justify the relationship	**1 point**

Example Solution

Energy in a torsion pendulum is conserved, $\frac{1}{2}I\omega^2 = \frac{1}{2}\kappa\theta^2$, *where* $\frac{1}{2}\kappa\theta^2$ *is a constant.*

Therefore, since $I_U < I_{non\text{-}U}$, $\omega_U > \omega_{non\text{-}U}$ *to keep energy conserved.*

Total for part (d) 5 points

Total for question 2 15 points