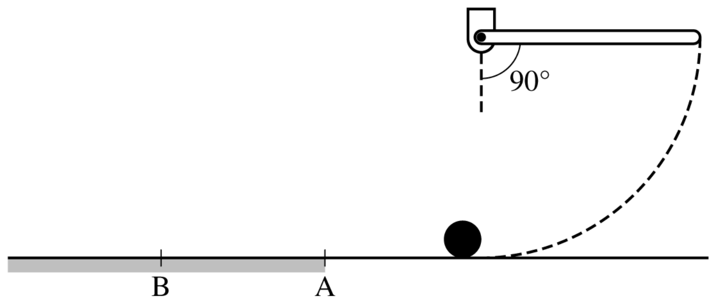
Begin your response to **QUESTION 3** on this page.




Note: Figure not drawn to scale.

Figure 1

3. A system consists of a small sphere of mass $m$ and radius $R$ at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length $\ell$ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length $\ell$ and physical constants as appropriate.

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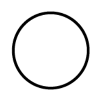
Continue your response to **QUESTION 3** on this page.

(b) Derive an expression for the linear speed $v_0$ of the sphere immediately after colliding with the rod in terms of the length $\ell$ and physical constants as appropriate.

After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction $\mu$, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.



(d) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of $v_0$, $\mu$, $R$, $t$, and physical constants as appropriate.

i. The linear velocity $v$ of the center of mass of the sphere as a function of time $t$

ii. The angular velocity $\omega$ of the sphere as a function of time $t$

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Continue your response to **QUESTION 3** on this page.

(e)

i. Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of $v_0$, $\mu$, and physical constants as appropriate.

ii. Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of $v_0$.

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## Question 3: Free-Response Question — 15 points

**(a)** For using a correct expression for conservation of energy of the rod-Earth system **1 point**

**Example Response**

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\mathrm{CM}}\right) + \left(\frac{1}{2}I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\mathrm{cm}} = \frac{1}{2}I\omega_f^2$$

For correctly substituting $h$ and $I$ into the correct energy expression **1 point**

**Example Response**

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

**Example Solution**

$$\Delta U + \Delta K = 0$$

$$\left(0 - Mgh_{\mathrm{CM}}\right) + \left(\frac{1}{2}I\omega_f^2 - 0\right) = 0$$

$$Mgh_{\mathrm{cm}} = \frac{1}{2}I\omega_f^2$$

$$Mg\frac{\ell}{2} = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega_f^2$$

$$\therefore \omega_f = \sqrt{\frac{3g}{\ell}}$$

| | | |
|---|---|---|
| **(e)(i)** | For indicating the linear speed is equal to $R\omega$ when slipping stops at Point B | **1 point** |

**Example Response**

$$v = R\omega$$

For correctly substituting $v$ and $\omega$ from parts (d)(i) and (d)(ii) **1 point**

**Scoring Note:** Substituting the acceleration from part (d)(i) into a valid kinematic equation that includes time can earn this point.

**Example Response**

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

**Example Solution**

$$v = R\omega$$

$$v_0 - \mu g t = R\frac{5\mu g}{2R}t$$

$$\therefore t = \frac{2v_0}{7\mu g}$$

| | | |
|---|---|---|
| **(e)(ii)** | For correctly substituting the expression for time from (e)(i) into the expression for velocity in (d)(i) | **1 point** |

**Example Solution**

$$v = v_0 - \mu g t$$

$$v = v_0 - \mu g\frac{2v_0}{7\mu g}$$

$$\therefore v = \frac{5}{7}v_0$$

**Scoring Note:** The last equation is not needed for scoring the item but is presented for clarity.

**Total for part (e) 3 points**

**Total for question 3 15 points**