## Question 4

**4.** A uniform disk and ring, each of mass $M$ and radius $R$, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed $v$.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle $\theta$ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

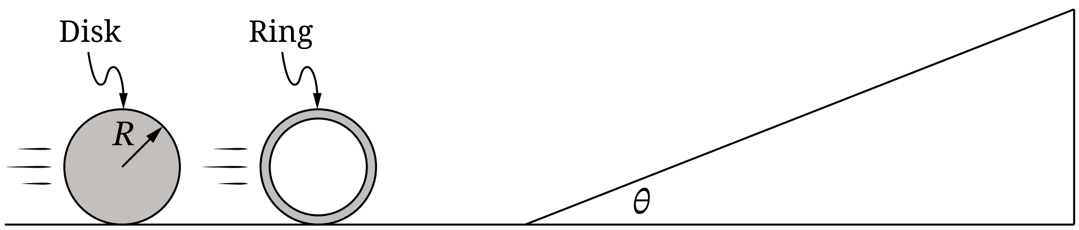


Figure 1

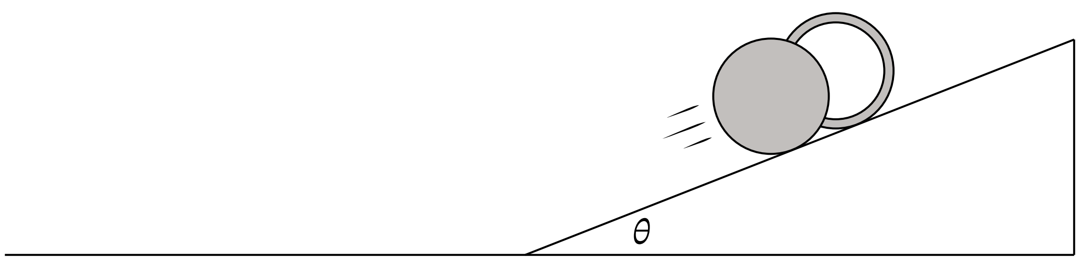


Figure 2

**A.** While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are $f_D$ and $f_R$, respectively.

**Indicate** whether $f_D$ is greater than, less than, or equal to $f_R$ by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

**Justify** your answer using qualitative reasoning beyond referencing equations.

**B.** A cylinder has mass $M$, radius $R$, and rotational inertia $I$ about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle $\theta$ above the horizontal.

**Derive** an expression for the magnitude of the static frictional force $f$ exerted on the cylinder by the ramp. Express your answer in terms of $M$, $R$, $I$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

**C.** In a different scenario, the centers of mass of the original disk and ring each have the same initial speed $v$ as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

**Indicate** whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

**STOP**

**END OF EXAM**

## Question 4: Qualitative Quantitative Translation (QQT) — 8 points

**A**

For indicating $f_D < f_R$ — **Point A1**

For a justification that compares **one** of the following: — **Point A2**

- The motions of the disk and the ring using translational kinematics
- The motions of the disk and the ring using rotational kinematics
- The rotational inertias of the disk and the ring

For a justification that includes **one** of the following: — **Point A3**

- Reasoning that attempts Newton's second law in translational form
- Reasoning that attempts Newton's second law in rotational form
- Reasoning that attempts conservation of energy

**Example Response**

*The ring has a greater rotational inertia because it has more mass distributed towards the edge. So the ring travels farther and has less acceleration down the ramp. Because the ring and the disk have the same mass, the gravitational forces exerted on the shapes are the same. From Newton's second law, the ring must have less net force down the ramp and more friction up the ramp.*

**B**

For a multistep derivation that includes Newton's second law in both translational and rotational forms — **Point B1**

For indicating opposite signs for the gravitational force and frictional force in an expression for Newton's second law — **Point B2**

For using the relationship $a = r\alpha$ in an attempt to solve a system of equations — **Point B3**

**Scoring Note:** Responses that include conservation of energy may earn full credit.

**Example Response**

$$\alpha_{sys} = \frac{\sum \tau}{I_{sys}}$$

$$\left(\frac{a}{R}\right) = \frac{fR}{I}$$

$$a = \frac{fR^2}{I}$$

$$\vec{a}_{sys} = \frac{\sum \vec{F}}{m_{sys}}$$

$$f - Mg\sin\theta = -Ma$$

$$f - Mg\sin\theta = -M\frac{fR^2}{I}$$

$$f + M\frac{fR^2}{I} = Mg\sin\theta$$

$$f\left(1 + \frac{MR^2}{I}\right) = Mg\sin\theta$$

$$f = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}}$$