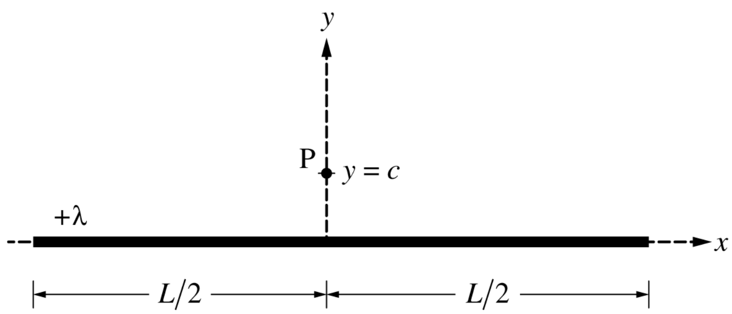
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

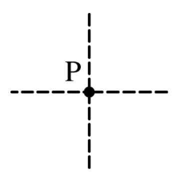


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

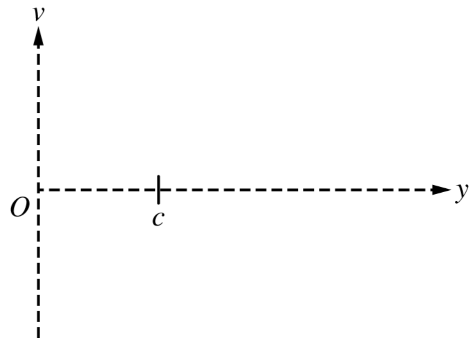
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

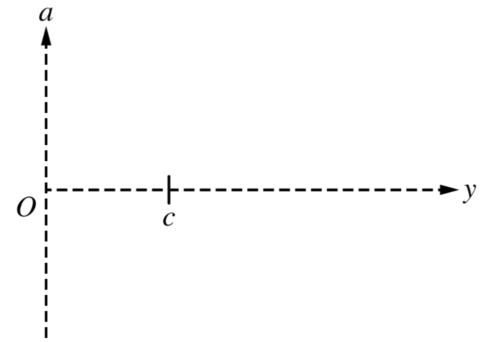


ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

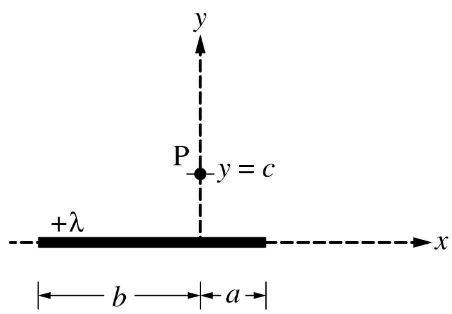
iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

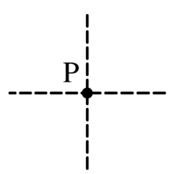




The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

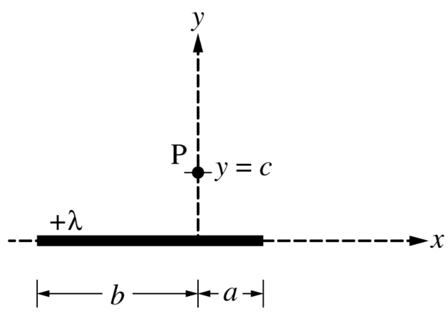


(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\,dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$\left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{x}{\left(c^2 + x^2\right)^{3/2}}\,dx$$

Vertical component: $$\left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0}\int_{-b}^{a}\frac{y}{\left(c^2 + x^2\right)}\,dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component   ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

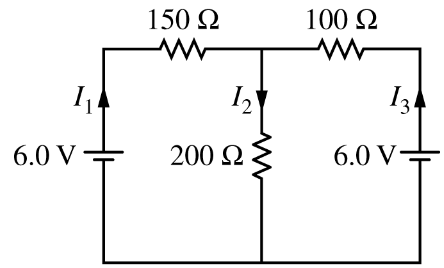


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

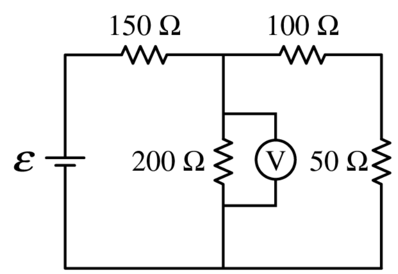


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

(c) Calculate the voltage $\mathcal{E}$ of the battery.