Begin your response to **QUESTION 1** on this page.

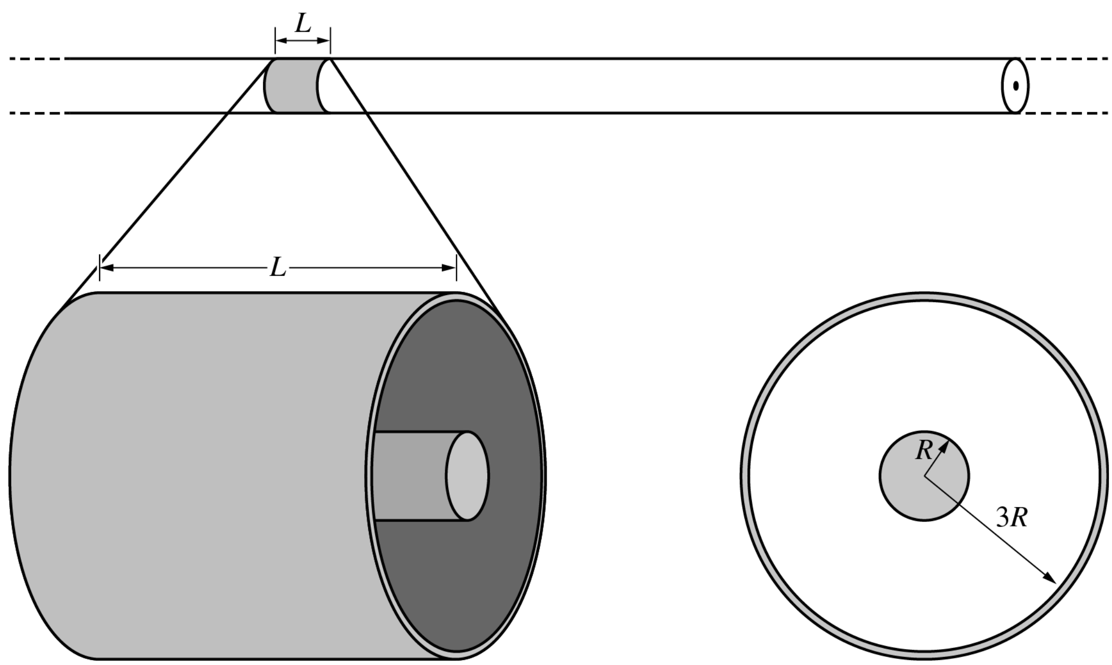
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View

Cross-Section View

Note: Figures not drawn to scale.

1. A very long nonconducting cylinder is surrounded by a thin concentric conducting cylindrical shell, as shown in the cutout view. A segment of length L of the inner cylinder has a net charge of $+Q$ uniformly distributed throughout its volume. A segment of length L of the outer shell has a net charge of $+4Q$. The radii of the inner cylinder and outer shell are R and $3R$, respectively, as shown in the cross-section view.

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Continue your response to **QUESTION 1** on this page.

(a) Determine the charge on the outer surface of the cylindrical shell within length L.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the inner cylinder for $r < R$. Express your answers in terms of Q, R, r, L, and physical constants, as appropriate.

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Continue your response to **QUESTION 1** on this page.

(c) The magnitude of the electric field at $r = R$ is $12\ \text{N}/\text{C}$. Calculate the value of the electric field at $r = 2R$.

(d) Derive an expression for the absolute value of the potential difference between the surface of the nonconducting cylinder and the inner surface of the cylindrical shell.

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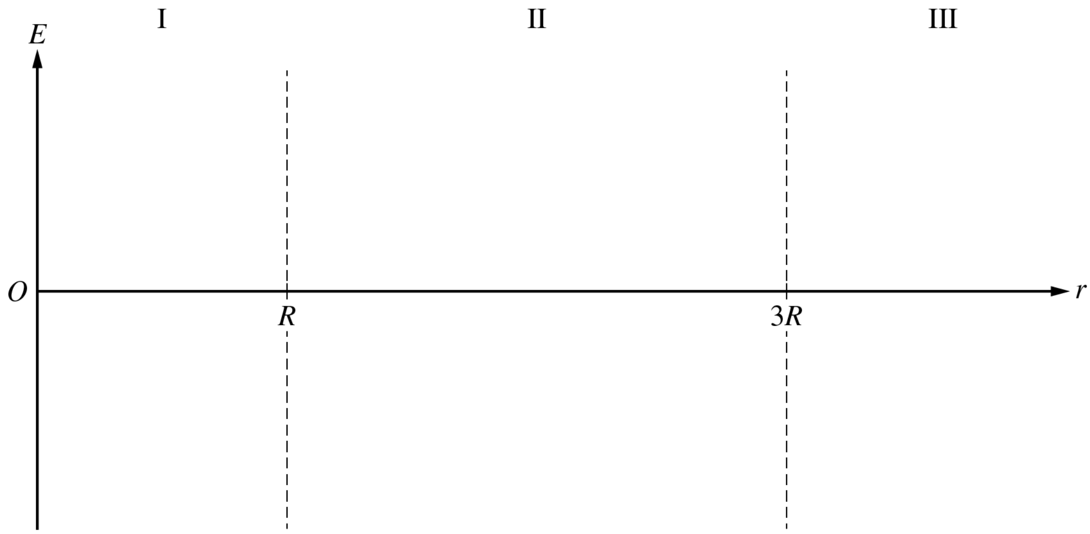
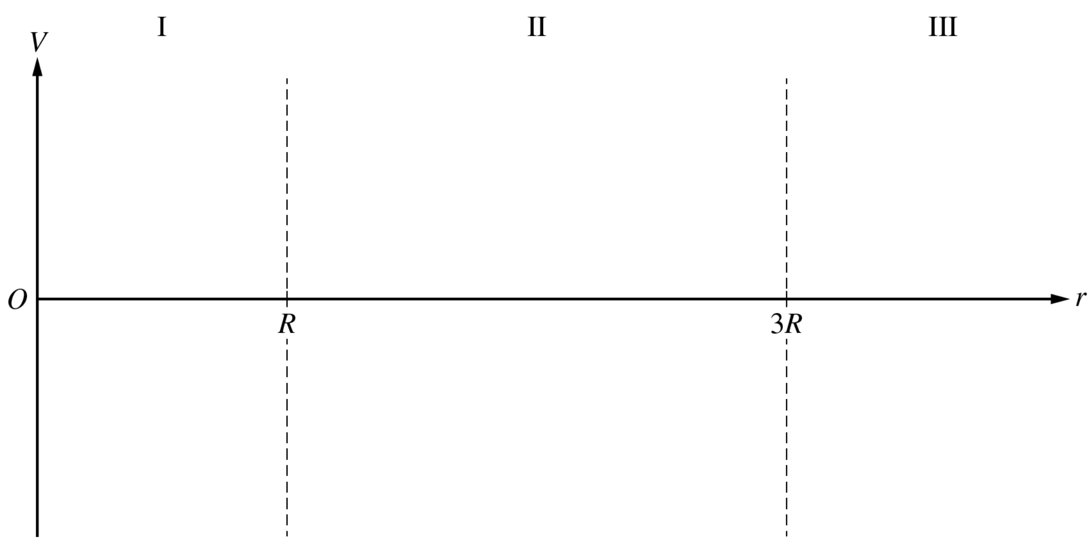
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch the graph of the electric field E as a function of the distance r from the axis of the inner cylinder.



ii. On the following axes that include regions I, II, and III, sketch the graph of the electric potential V as a function of the distance r from the axis of the inner cylinder.



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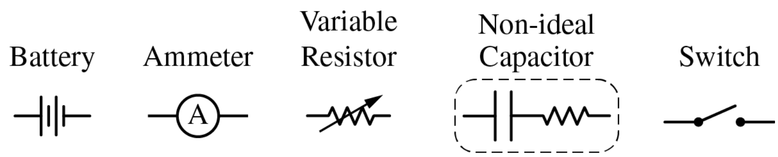
Begin your response to **QUESTION 2** on this page.

2. A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the current through the capacitor as it discharges through the resistor.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor.

(b) Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

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Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +4Q - (-Q)$$
$$q_{outer} = +5Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — **1 point**

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E(2\pi rL)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)\left(\pi r^2 L\right) = Q\frac{r^2}{R^2}$$

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0 (2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0 (2\pi rL)}$$
$$E = Q\frac{r}{2\pi\varepsilon_0 R^2 L}$$

Total for part (b) — **3 points**