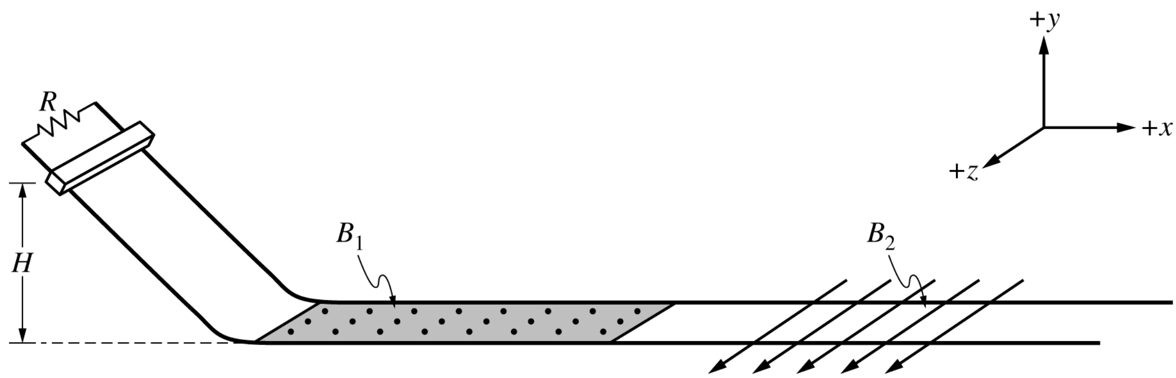
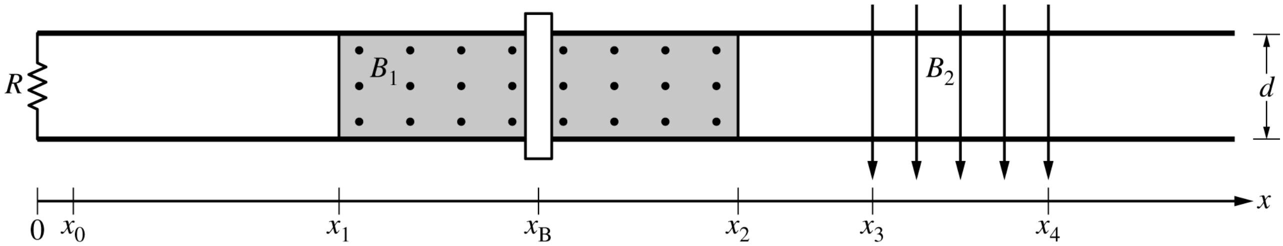
Begin your response to **QUESTION 2** on this page.



Perspective View (at time $t = 0$)



Top View (at time t_B)

2. Two parallel conducting rails are separated by distance $d = 0.30\text{ m}$. A resistor of resistance $R = 0.20\ \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

   - At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
   - At time t_1, the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40\text{ T}$ that is directed in the $+y$-direction.
   - At time t_2, the bar reaches position x_2 and enters a region with no magnetic field.
   - At time t_3, the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60\text{ T}$ that is directed in the $+z$-direction.
   - At time t_4, the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

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Continue your response to **QUESTION 2** on this page.

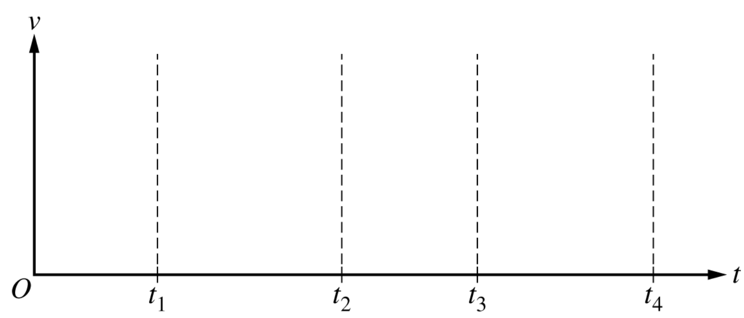
(a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B. If the net force is zero, write $F_{net} = 0$.

(b) At time t_B, the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B.

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B.

(c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4.



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Continue your response to **QUESTION 2** on this page.

(d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20\,\Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.

i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original}. In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new}. Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

(e) Describe a modification to H, B_1, or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

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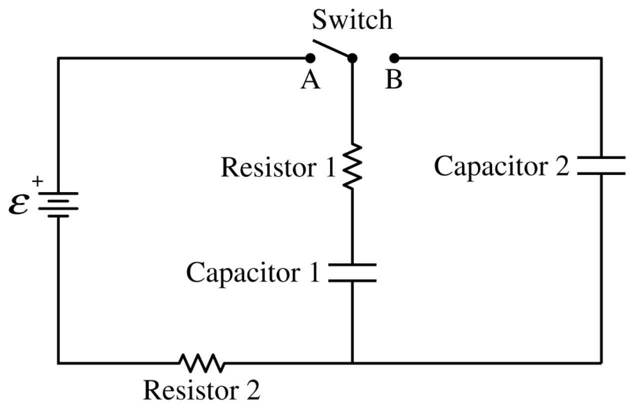
Begin your response to **QUESTION 3** on this page.



3. The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

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(e)	For correctly indicating **one** of the following, with an attempt at a relevant justification: • Increasing H • Increasing B_1 • Increasing d	**1 point**
	For correctly justifying the identified modification that will result in a larger induced current in the new bar at position x_B	**1 point**

Example Responses

Increasing H will increase the induced current. If the ramp is higher, then the potential energy is greater and this results in greater kinetic energy and greater velocity at the bottom of the ramp. A greater velocity causes a greater rate of change in flux as the bar moves through the field. By Faraday's law the emf *is greater and therefore also the current.*

OR

Increasing B_1 will increase the induced current. If the magnetic field is stronger this increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf *is then greater and therefore also the current.*

OR

Increasing d will increase the induced current. A larger width results in a greater area encompassed by the circuit. Greater area increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf *is then greater and therefore also the current.*

Total for part (e) 2 points

Total for question 2 15 points

(c)(i) For a correct justification that could include one of the following: **1 point**

- An indication that the current is upwards that includes a statement that indicates that positive charge had accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
- An indication that the current is upwards that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A

Example Responses

When the switch is closed at time t_1, positive charge has built up on the top plate of the capacitor. This positive charge on the top plate pushes charge up through Resistor 1 and down through Capacitor 2 to charge Capacitor 2.

OR

After a long-time the top plate of Capacitor 1 is at a high potential due to its being charged by the battery. When the switch is closed at time t_1, the resulting current is up through Resistor 1 as the current goes from high potential on the top plate to low potential clockwise around the circuit through Capacitor 2.

(c)(ii) For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ **1 point**

Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has the same capacitance of Capacitor 1. Therefore, Capacitor 2 stores the same charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores half of the original charge equal to $\frac{Q_0}{2}$.

(c)(iii) For indicating that the total energy dissipated by Resistor 1 is the difference between the initial electric potential energy of the system at time $t = t_1$ and the final electric potential energy of the system after the new steady state conditions have been reached **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$

For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store electric potential energy after the new steady-state conditions have been reached, or an alternate response that is consistent with part (c)(ii) **1 point**

Example Response

$$U_{0C} = U_{01} \textbf{ AND } U_C = U_1 + U_2$$

For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached based on part (c)(ii) **1 point**

Example Solution

$$\Delta E_R = U_C - U_{0C}$$

$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{2}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$

$$\Delta E_R = \frac{Q_0^2}{4C} - \frac{1}{2}\frac{Q_0^2}{C}$$

$$\Delta E_R = -\frac{Q_0^2}{4C}$$

Total for part (c) **5 points**

(d) For the correct expression for charge on the top plate of Capacitor 2 $\left(Q_2 = \frac{2Q_0}{3}\right)$ **1 point**

Example Response

$$C_1 = C \text{ and } C_2 = \kappa C = 2C$$

$$\Delta V_{C1} = \Delta V_{C2}$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = \frac{1}{2}Q_2$$

$$Q_1 + Q_2 = Q_0$$

$$\frac{1}{2}Q_2 + Q_2 = Q_0$$

$$Q_2 = \frac{2}{3}Q_0$$

Scoring Note: This point can be earned without supporting calculations.

Total for part (d) **1 point**