Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

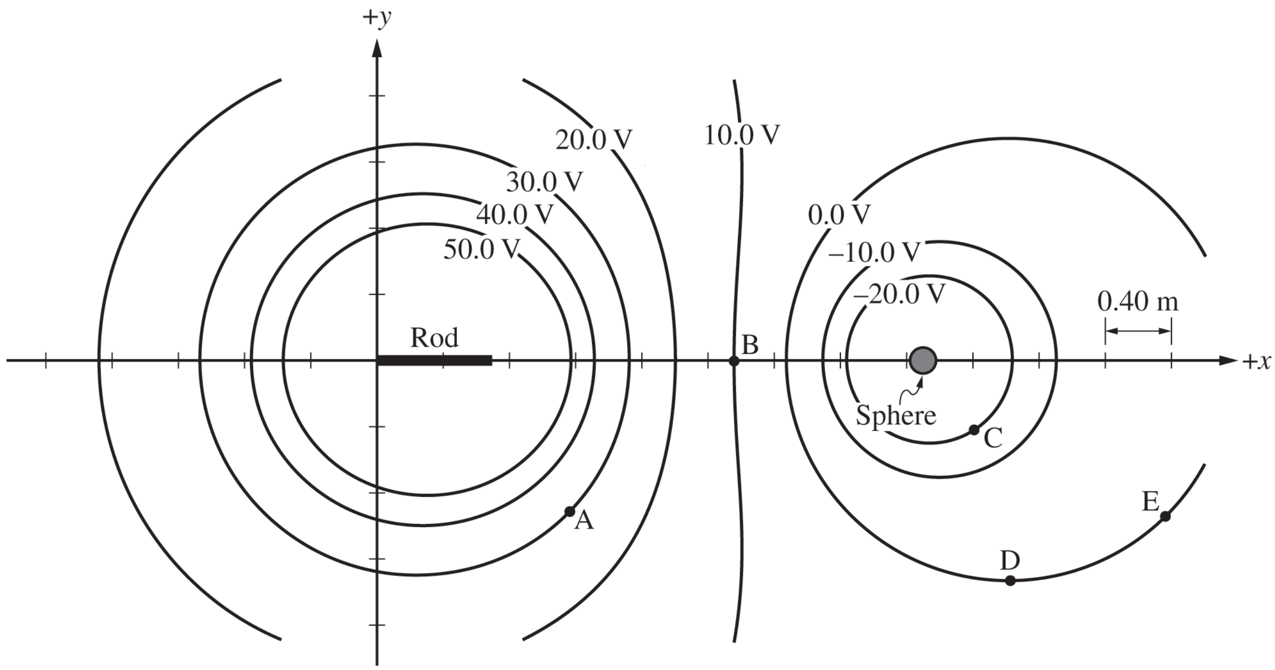

Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC. The rod and sphere are held at rest on the x-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

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A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

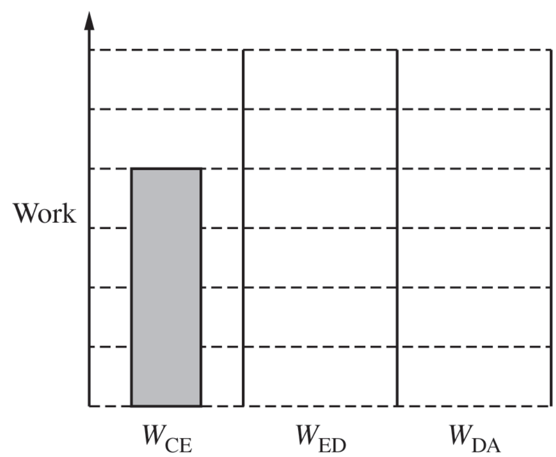

Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

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The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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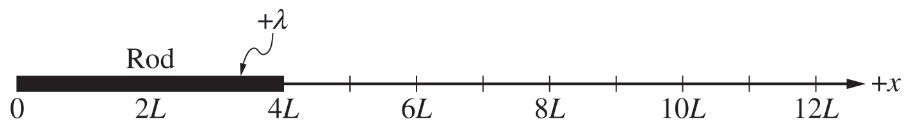

Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x-axis in the orientation shown in Figure 3. Position P (not shown) is located on the x-axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\dfrac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

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ii. On Figure 4, **sketch** a graph of the x-component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

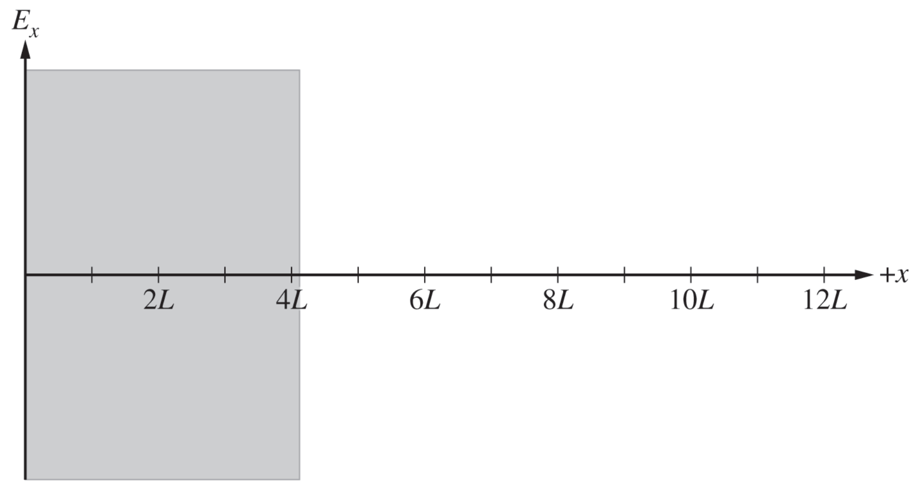

Figure 4

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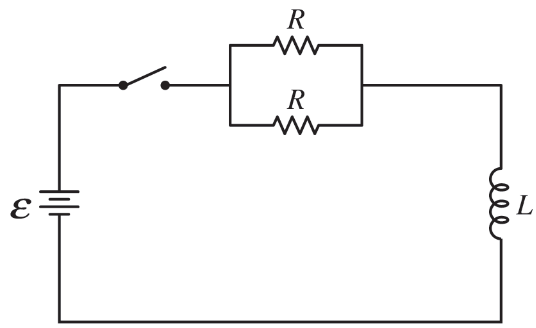

Figure 1

2. Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.

Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

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Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer **1 point**

Scoring Note: This point can be earned if a negative sign is included in the final answer or if units are missing and/or the units are incorrect.

Example Response

$$|\Phi_E| = 226 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{-2.0 \times 10^{-9}\ \text{C}}{8.85 \times 10^{-12} \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}}$$

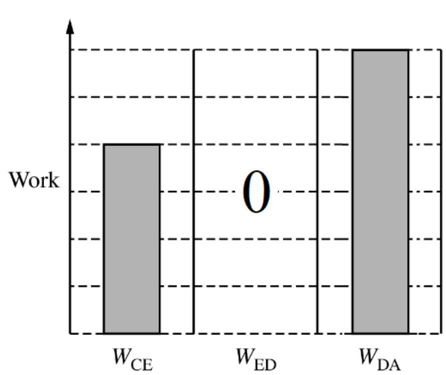
$$|\Phi_E| = 226 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Total for part (a) 2 points

(b)(i) For indicating that $W_{\text{ED}} = 0$ **1 point**

For drawing a bar representing W_{DA} that has a height of six units **1 point**

Example Response



(b)(ii)	For using an equation that relates the electric field to potential difference	**1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$E_x = -\frac{dV}{dx}$ **OR** $|E_x| = \left|-\frac{dV}{dx}\right|$ **OR** $|E_x| = \left|-\frac{\Delta V}{\Delta x}\right|$ **OR** $\Delta V = -\int \vec{E} \cdot d\vec{r}$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$$|E_x| = \left|-\frac{20.0\text{ V} - 0.0\text{ V}}{0.65\text{ m}}\right|$$

Example Solution

$$E_x = -\frac{dV}{dx}$$

$$|E_x| = \left|-\frac{dV}{dx}\right|$$

$$|E_x| = \left|-\frac{\Delta V}{\Delta x}\right|$$

$$|E_x| = \left|-\frac{20.0\text{ V} - 0.0\text{ V}}{0.65\text{ m}}\right|$$

$$|E_x| = 31\,\frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c)	For selecting only $+y$ with an attempt at a relevant justification	**1 point**
	For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position D	**1 point**
	For indicating one of the following:	**1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the sphere have charges of opposite sign.
- The test charge moves in the direction of the electric field, which is directed upward.

Example Response

$+y$. *The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position* D , *the electric force is upward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.*

Total for part (c) 3 points

(d)(i)	For using an appropriate equation for determining the electric potential from a line of uniform charge	**1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r , the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_\mathrm{P} = k\sum\frac{Q}{x_\mathrm{P} - x} \quad \textbf{OR} \quad V_\mathrm{P} = k\int\left(\frac{1}{x_\mathrm{P} - x}\right)dq$$

For a correct integral with λdx substituted for dq **1 point**

Example Response

$$V_\mathrm{P} = k\lambda\int\left(\frac{1}{x_\mathrm{P} - x}\right)dx$$

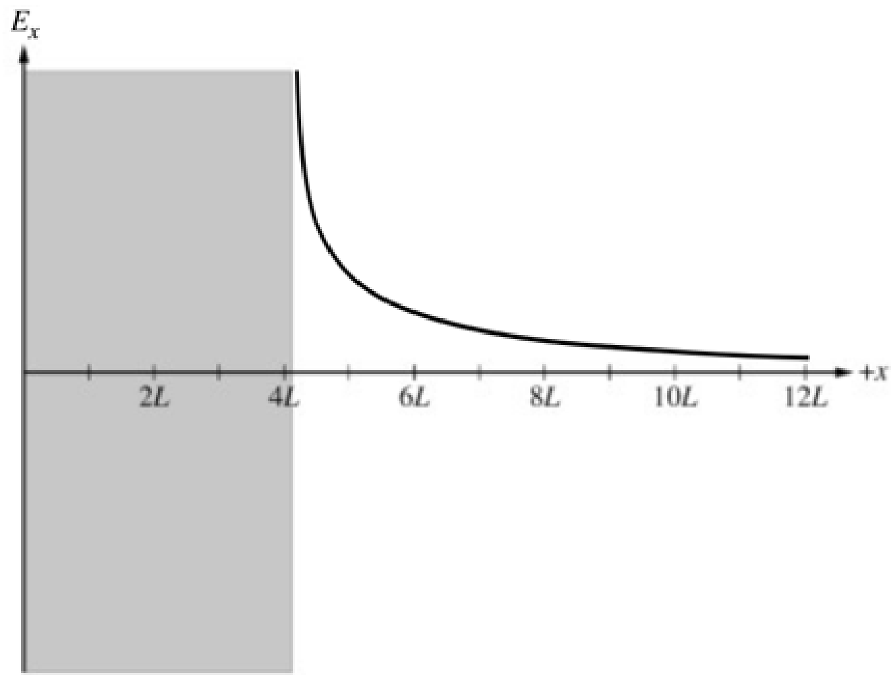
For the correct limits of integration **1 point**

Example Response

$$V_\mathrm{P} = k\lambda\int_0^{4L}\left(\frac{1}{x_\mathrm{P} - x}\right)dx$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave up curve that is always positive	**1 point**

Example Response



Total for part (d)	**6 points**
Total for question 1	**15 points**