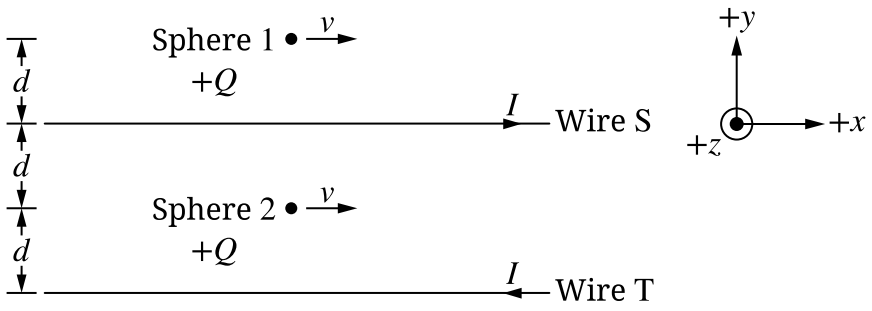
Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

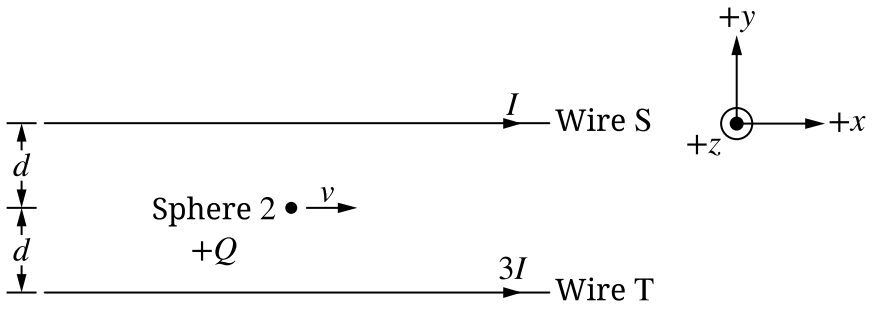
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP

END OF EXAM

Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A

For indicating that $F_2 > F_1$ — **Point A1**

For correctly relating the magnitude of the magnetic force to the magnitude of the magnetic field — **Point A2**

For indicating that the magnitude of the magnetic field at the location of Sphere 2 is greater than the magnitude of the magnetic field at the location of Sphere 1 by referring to either: — **Point A3**

- The relative distance from the spheres to Wires S and T
- The direction of the magnetic fields from Wires S and T at the locations of the spheres

Example Response

The magnitude of the magnetic force exerted on a sphere is directly proportional to Q, v, and B. Because Q and v are the same for both spheres, the difference in the magnitudes of the magnetic forces is due to the difference in the magnitudes of the magnetic fields at the locations of the spheres. The magnitude of the magnetic field is greater at the location of Sphere 2 than at the location of Sphere 1 because the fields from the wires are in the same direction. Therefore, $F_2 > F_1$.

B

For a multistep derivation that includes the equation $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ or — **Point B1**

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\left(d\vec{\ell} \times \hat{r}\right)}{r^2}$$

Scoring Note: Vector notation is not required for this point to be earned. A multistep derivation that includes $B = \frac{\mu_0 I}{2\pi d}$ can earn this point.

For a correct expression for the magnitude of the magnetic field due to the current in one wire at the location of Sphere 2 — **Point B2**

For indicating that the magnitude of the total field is twice that due to one wire — **Point B3**

Example Response

Determine the magnitude B of the magnetic field due to one wire.

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$

$B(2\pi d) = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi d}$

Determine B_{tot}.

$B_{\text{tot}} = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi d}$

$B_{\text{tot}} = \frac{\mu_0 I}{\pi d}$

C	For indicating $F_{\text{new}} = F_2$	**Point C1**
	For indicating $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d}$ OR For indicating that the net magnitude of the magnetic field at the location of Sphere 2 will remain the same even though the field from Wire T will change	**Point C2**
	Example Response *The magnetic field at the location of Sphere 2 due to the current in Wire T is in the opposite direction to the magnetic field due to the current in Wire S. The current in Wire T is increased by a factor of 3. Therefore,* $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d}$ *in this scenario. Thus,* $F_{\text{new}} = F_2$.	