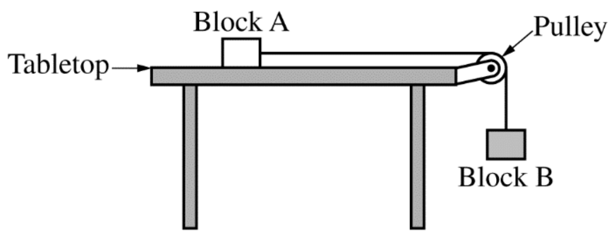


2. (12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much <u>greater</u> than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much <u>less</u> than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Block A

Block B

(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much <u>less</u> than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

____ Yes ____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much <u>less</u> than m_B.

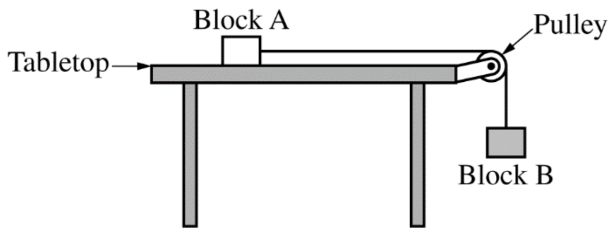
(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

____ $T_2 > T_1$ ____ $T_2 = T_1$ ____ $T_2 < T_1$

Briefly explain your reasoning.

Question 2

12 points



This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a) LO 3.A.1.1, SP 1.5; LO 3.B.1.1, SP 6.4, 7.2

i.
2 points

Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

Examples of correct answers: "Zero", "small", "negligible", "much less than g", or "<<g"		
For a correct answer and attempt at a consistent justification		1 point
For correct reasoning		1 point
Example earning 1 point: Nearly zero. Because block A is much heavier than block B.		
Examples earning 2 points: "Very small. Because block A has a large inertia, it won't speed up much." "Close to zero because block B is so light that it can hardly budge block A."		
Claim: The acceleration of the blocks is zero/small/negligible/ "<<g". Evidence: The mass of block A is much greater than the mass of block B. Reasoning: See two-point examples above.		

ii.
1 point

Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

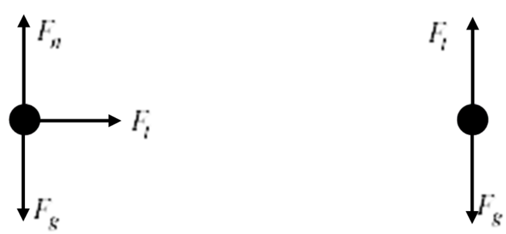
Question 2 (continued)

(a) (continued)
ii. (continued)

Examples of correct answers: g or 9.8 m/s^2 or 10 m/s^2 (or just 9.8 or 10)		
For a correct answer and correct justification		1 point
Examples: Nearly equal to g. Because block B is almost in free fall. 10 m/s^2, because block A has negligible mass and the tension in the string is nearly zero.		
Claim: The acceleration of the blocks is close to g. Evidence: The mass of block A is much less than the mass of block B. There is negligible friction between block A and the tabletop. The pulley has negligible mass and spins with negligible friction about its axle. Reasoning: See examples above.		

(b) LO 3.A.2.1, SP 1.1; LO 3.A.3.1, SP 6.4
3 points

Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



Block A Block B

For a correct normal force on block A with acceptable label: N, F_N, "normal force," F_{table}, "table force," or any other label indicating the force is "normal" or comes from the table		1 point
For correct gravitational forces with acceptable label on both diagrams: F_g, F_{grav}, W, mg, $m_A g$, "gravity," "grav force," but NOT G or g, and no extraneous forces on either diagram		1 point
For correct tension forces with acceptable label on both diagrams: "tension," "string force," F_T, $F_{tension}$, F_{string}, F_S, T, or some other label indicating that the force comes from the string or from tension. NOT acceptable: $m_B g$, F_{m_B}, "force from block B" or other indications that the force is "created" by block B		1 point

Question 2 (continued)

(c) LO 2.B.1.1, SP 2.2; LO 3.A.1.1, SP 1.5, 2.2; LO 3.B.1.3, SP 1.5, 2.2; LO 3.B.2.1, SP 1.4, 2.2; LO 4.A.2.1, SP 6.4
3 points

Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using separate Newton's second law equations for each block		1 point
For combining the equations with correct notation, including correctly using m_A and m_B, indicating that the same tension force acts on both blocks, *and* that they share the same acceleration		1 point
For a correct equation for a with supporting work: $a = \dfrac{m_B}{m_A + m_B} g$		1 point

Alternate Solution:

For writing a "whole-system" equation for the total mass that does not contain internal forces. $F_{\text{net}} = m_{\text{total}} a$		*1 point*
For substituting the net force and system mass with correct quantities $m_B g = (m_A + m_B) a$		*1 point*
<u>Note:</u> *Writing the correct whole-system equation is sufficient to earn the first two points.*		
For a correct equation for a with supporting work: $a = \dfrac{m_B}{m_A + m_B} g$		*1 point*

(d) LO 3.A.1.1, SP 2.2; LO 3.A.3.1, SP 6.4; LO 3.B.1.3, SP 2.2
1 point

Consider the scenario from part (a)(ii), where the mass of block A is much <u>less</u> than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

____ Yes ____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much <u>less</u> than m_B.

Correct answer: "Yes"		
<u>Note:</u> "No" is acceptable if the equation is inconsistent with the answer in (a)(ii).		
For valid reasoning that addresses the result in part (c) and the reasoning in part (a)(ii)		1 point

Question 2 (continued)

(d) (continued)

Claims: Yes, the equation for the acceleration of the blocks from part (c) agrees with the reasoning in part (a)(ii). or No, the equation for the acceleration of the blocks from part (c) does not agree with the reasoning in part (a)(ii). Evidence: The mass of block A is much less than the mass of block B. $a = \frac{m_B}{m_A + m_B} g$ (derived as part (c) answer) Reasoning for "Yes" claim: When m_A is much less than m_B, it can be neglected in the equation derived in part (c), giving an acceleration close to g as stated in (a)(ii). Reasoning for "No" claim, if the answer in part (a)(ii) is wrong: When m_A is much less than m_B, it can be neglected in the equation derived in part (c), giving an acceleration close to g. This disagrees with the value of ___ stated in (a)(ii). Reasoning for "No" claim, if the answer in part (c) is wrong: When m_A is much less than m_B, it can be neglected in the equation derived in part (c), giving an acceleration of ___. This disagrees with the value of g stated in (a)(ii).		

(e) LO 3.A.1.1, SP 2.2; LO 3.B.1.1, SP 6.4,7.2; LO 3.B.1.3, SP 2.2
2 points

While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

____ $T_2 > T_1$ ____ $T_2 = T_1$ ____ $T_2 < T_1$

Briefly explain your reasoning.

Correct answer: $T_2 > T_1$. Note: A maximum of 1 point can be earned if an incorrect selection is made.		
For reasoning that the acceleration of both blocks is smaller		1 point
For doing any one of the following, consistent with the answer selection and Newton's second law for block B • Concluding that a smaller acceleration implies that T_2 is greater than T_1 • Concluding that an unchanged acceleration implies that T_2 is the same as T_1 • Concluding that a larger acceleration implies that T_2 is less than T_1		1 point

Question 2 (continued)

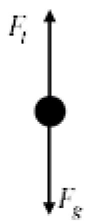
(e) (continued)

Claim: $T_2 > T_1$

Evidence:

The pulleys spin with negligible friction about the axle.
The original pulley has negligible mass.
The second pulley's mass is not negligible.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$



Block B

Reasoning:

The rotational inertia of the second pulley results in a smaller acceleration for the blocks. Block B must have a smaller net force to have a smaller acceleration, so the rope tension must be larger than before (closer in magnitude to the gravitational force on block B).

Learning Objectives

LO 2.B.1.1: The student is able to apply F = mg to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. [See Science Practices 2.2, 7.2]
LO 3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [See Science Practices 1.5, 2.1, 2.2]
LO 3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [See Science Practice 1.1]
LO 3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [See Science Practices 6.4, 7.2]
LO 3.B.1.1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. [See Science Practices 6.4, 7.2]
LO 3.B.1.3: The student is able to reexpress a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. [See Science Practices 1.5, 2.2]
LO 3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [See Science Practices 1.1, 1.4, 2.2]
LO 4.A.2.1: The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. [See Science Practice 6.4]