Begin your response to **QUESTION 3** on this page.

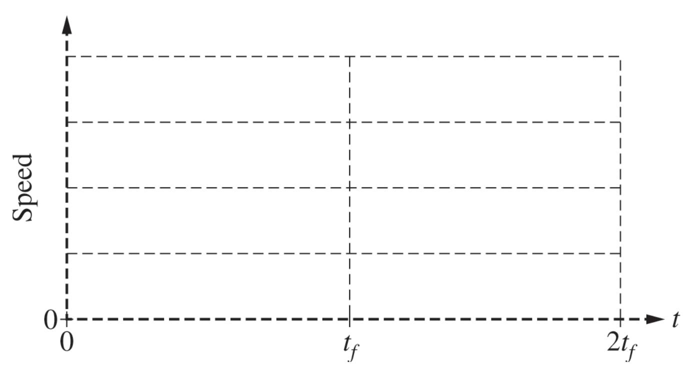
3. (12 points, suggested time 25 minutes)

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

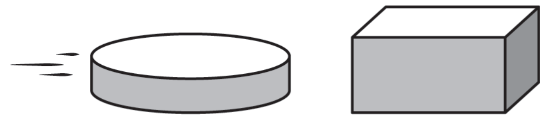
(b) Assume that the student's mass is greater than that of the disk $(M_S > M_D)$. On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.



GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Continue your response to **QUESTION 3** on this page.



(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

____ Yes ____ No

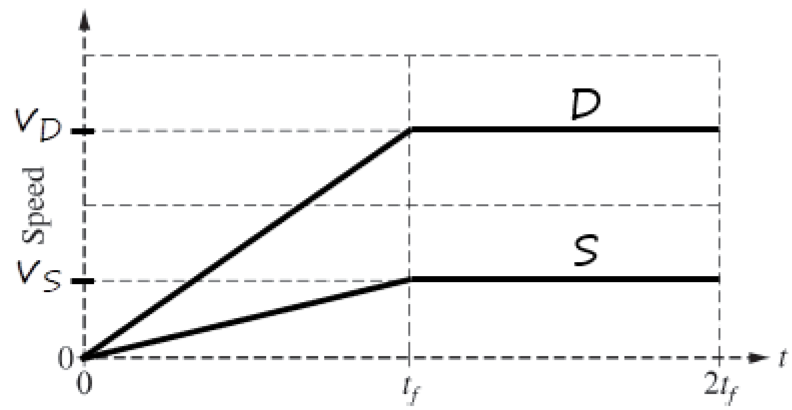
Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

GO ON TO THE NEXT PAGE.

Question 3: Qualitative/Quantitative Translation — 12 points

(a)	**i.**	For a correct answer $v_D = \frac{F_H t_f}{M_D}$	**1 point**
	ii.	For indicating the total momentum of the system is the same before and after the collision **Scoring Note**: If the response only includes a correct final answer of $\frac{M_S}{M_D}$, the response earns this point but not the next point.	**1 point**
		For correctly substituting the appropriate variables into a conservation of momentum equation **AND** an answer in the form $\frac{v_D}{v_S} = \ldots$ **Scoring Notes:** This point can be earned only if the first point is earned. The answer need not be correct to earn this point.	**1 point**
		Example response for part (a)(ii) $p_i = p_f$ $0 = M_S v_S - M_D v_D$ $\frac{v_D}{v_S} = \frac{M_S}{M_D}$	
		Total for part (a)	**3 points**
(b)		For two functions that are straight segments for $t < t_f$, **AND** begin at the origin, **AND** have two different positive slopes	**1 point**
		For two functions that are horizontal functions for $t > t_f$ **AND** are continuous over the entire time range $0 < t < 2t_f$	**1 point**
		For labeling values on the vertical axis with $v_D > v_S$ **OR** The curve labeled D is greater than the curve labeled S for all t > 0 **Scoring note:** This point can still be earned if the labels are not on the vertical axis but clearly indicate that $v_D > v_S$.	**1 point**

Example response for part (b)



		Total for part (b)	**3 points**
(c)	**i.**	For stating or mathematically representing that if the disk is much more massive, then the block will have little effect on the motion of disk 1 **OR** For stating or mathematically representing that when $M_D >> M_B$, $v_{cm} = v_1$	**1 point**
	ii.	For correct reasoning. Correct answer: When $M_D << M_B$, $v_{cm} = 0$	**1 point**
		Example response for part (c) (ii) *If the block is much more massive, then it will barely move when the disk collides and sticks to it.*	
	iii.	For using conservation of momentum	**1 point**
		For a correct answer $v_{cm} = \dfrac{m_D v_1}{m_D + m_B}$	**1 point**
	iv.	For an attempt to use limiting-case reasoning or functional dependence with the equation in part (c)(iii)	**1 point**
		For recognizing the equation from (c)(iii) reduces to a simpler form and the simplified form is correctly compared to their answer in (c)(i)	**1 point**
		Example 1 response for part (c) (iv) *Yes. If M_B is very small, then the denominator of the equation simplifies to M_D, which then can cancel out of the equation leaving $v_{cm} = v_1$.*	
		Total for part (c)	**6 points**
		Total for question 3	**12 points**