Begin your response to **QUESTION 5** on this page.

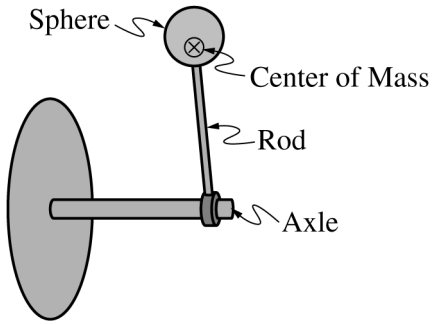

Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.
Note: these frames are not equally spaced apart in time.

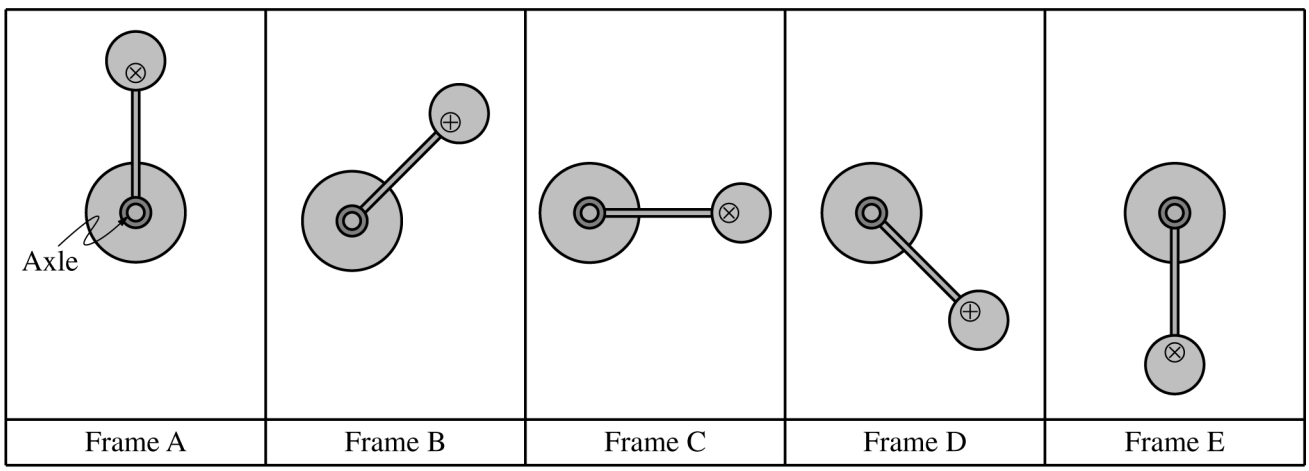

Figure 2

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 5** on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

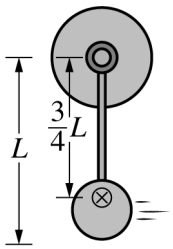

Figure 3

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

GO ON TO THE NEXT PAGE.

Question 5: Short Answer — 7 points

(a)(i) For indicating "Frame C" with correct reasoning about the magnitude of the torque being the greatest — **1 point**

Accept **one** of the following:

- This is the instant when the lever arm is greatest.
- This is when the angle between radius vector and weight force vector is most perpendicular.

For correctly relating torque and angular acceleration: $\alpha \propto \tau$ — **1 point**

Example Response

The angular acceleration is greatest in Frame C because angular acceleration is proportional to torque, and in Frame C the gravitational force vector is directed perpendicular to the rod (lever arm) which means this is where the torque will be the greatest.

(a)(ii) For indicating "Frame E" with correct reasoning — **1 point**

Accept **one** of the following:

- Work or energy (e.g., this is when the maximum work has been done on the system by gravity.)
- Angular momentum (e.g., the torque due to gravity is clockwise the entire time, causing the rod to gain angular momentum.)
- Kinematics (e.g., the rod speeds up the entire time.)

Example Response

The rotational kinetic energy is greatest in Frame E because this is where the rod-sphere system has the greatest rotational speed since the torque has been in the same direction as the motion the entire time.

Total for part (a) 3 points

(b)(i) For a multistep derivation that begins with conservation of energy **1 point**

$E_i = E_f$ **OR** $\Delta E = 0$ **OR** $U_{gi} + K_i = U_{gf} + K_f$

For indicating the change in height is equal to $\frac{3}{2}L$ **1 point**

$\Delta y = \frac{3}{2}L$

For an answer consistent with the height change indicated previously in the response **1 point**

$K_f = \frac{3}{2}MgL$

Scoring Note: A correct answer of $K_f = \frac{3}{2}MgL$ with no supporting work can earn only this point.

Example Response

$E_i = E_f$
$U_{gi} + K_i = U_{gf} + K_f$
$\Delta K = U_{gi} - U_{gf}$
$\Delta K = Mg\Delta y$
$\Delta y = \frac{3L}{4} + \frac{3L}{4} = \frac{3}{2}L$
$\Delta K = \frac{3}{2}MgL$

(b)(ii) For indicating that the gravitational force is the external force that does work on the rod-sphere system **1 point**

Example Response

The rod and sphere gain kinetic energy due to the positive work done by the gravitational force, which is an external force for the rod-sphere system.

Total for part (b) 4 points

Total for question 5 7 points