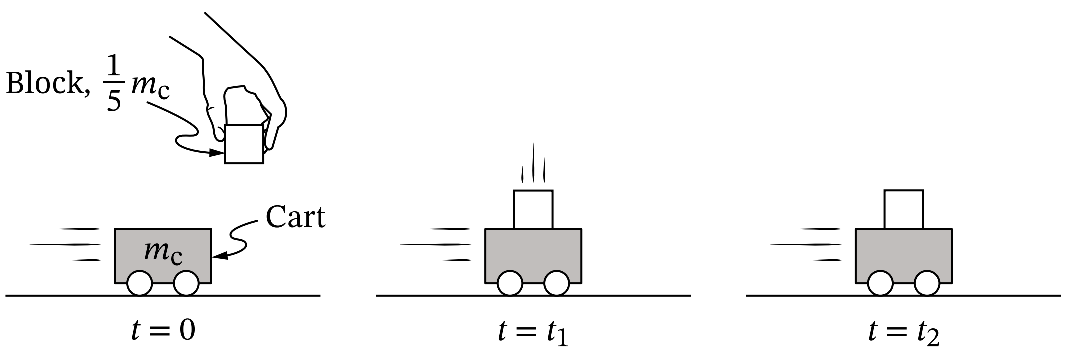
Question 1: Version J

1. A student has a cart of mass m_c and a block of mass $\frac{1}{5}m_c$, as shown in Figure 1.

- At time $t = 0$, the cart is moving to the right across a horizontal surface with constant speed v_c, and the student releases the block from rest.
- At $t = t_1$, the block collides with and sticks to the top of the cart. The block does not slide on the cart.
- At $t = t_2$, the block-cart system continues to move to the right with constant speed v_f.

Figure 1



A.

i. On the axes shown in Figure 2, **sketch** a graph of the magnitude p_x of the x-component of the momentum of the block-cart system as a function of time t from $t = 0$ until $t > t_2$.

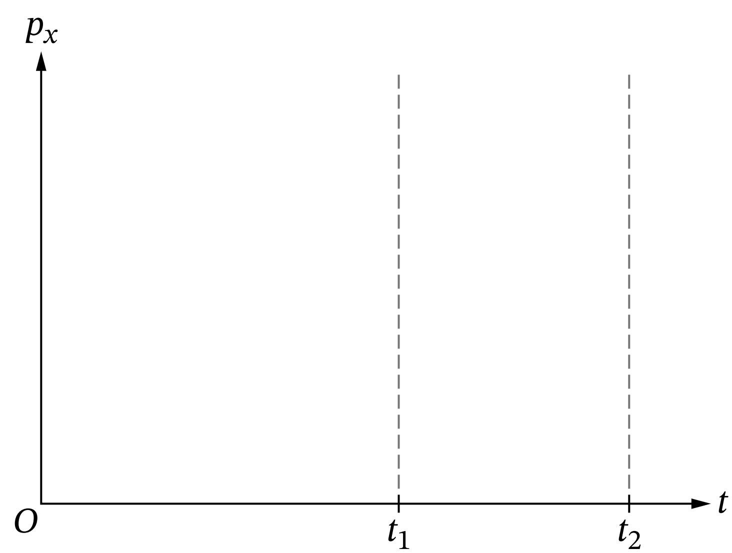


Figure 2

Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For sketching **one** of the following: **Point A1**

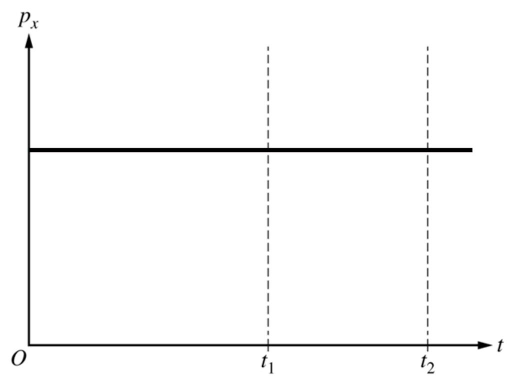
- A constant p_x for $t < t_1$
- A constant p_x for $t > t_1$

For sketching a line that demonstrates momentum is constant **Point A2**

Accept **one** of the following:

- A continuous, nonzero, horizontal line from $t = 0$ to $t = t_2$
- A continuous, nonzero, horizontal line from $t = 0$ to $t > t_2$

Example Response



(ii) For including a conservation of momentum equation **Point A3**

Scoring Note: Part A (ii) and part A (iii) may be scored together, if necessary.

For setting $m_c v_c$ equal to the momentum of the block-cart system after the collision **Point A4**

Scoring Note: A correct, isolated, final expression of $v_f = \frac{5}{6} v_c$ earns points A3 and A4.

Example Response

$$p_i = p_f$$

$$m_c v_c = (m_c + m_b) v_f = \left(m_c + \frac{1}{5} m_c\right) v_f$$

$$v_f = \frac{5}{6} v_c$$

(iii) For a multistep derivation that includes the correct relationship between kinetic energy, mass, and speed (i.e., $K = \frac{1}{2}mv^2$) **Point A5**

Scoring Note: The minimum requirement to earn this point is to show an expression of kinetic energy or change in energy that goes beyond the given equation on the reference information. For example, substituting quantities from the problem into $K = \frac{1}{2}mv^2$ or indicating the change in kinetic energy is $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ earns the point.

For an indication that m_c and $\frac{6}{5}m_c$ are the initial and final masses, respectively, of the objects moving horizontally **Point A6**

For including an expression for v_f consistent with part A (ii) **Point A7**

Scoring Note: A correct, isolated, final expression of $\Delta K = -\frac{1}{12}m_c v_c^{\,2}$ earns points A3, A4, A6, and A7.

Example Response

$$\Delta K = K_f - K_0$$

$$K_0 = \frac{1}{2}m_c v_c^{\,2}$$

$$K_f = \frac{1}{2}\left(\frac{6}{5}m_c\right)\left(\frac{5}{6}v_c\right)^2 = \frac{5}{12}m_c v_c^{\,2}$$

$$\Delta K = -\frac{1}{12}m_c v_c^{\,2}$$