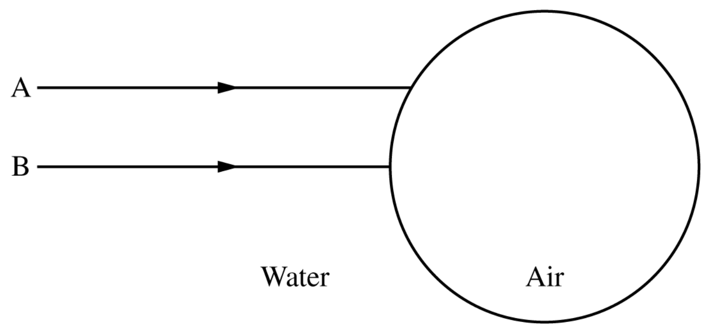
4. (10 points, suggested time 20 minutes)

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



(b) The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

(c) At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \text{kg/m}^3$.

i. Determine the absolute pressure in the bubble at this location.

ii. The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

iii. If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

STOP

END OF EXAM

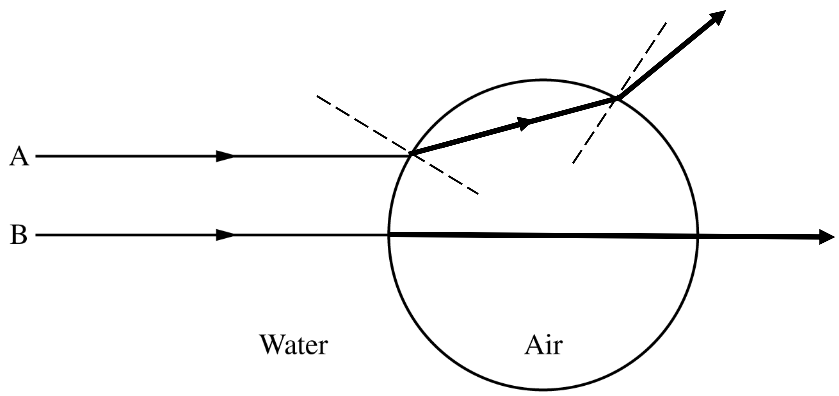
Question 4

10 points

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) LO 6.E.3.1, SP 1.1, 1.4
3 points

In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.



For ray B going straight through		1 point
For ray A bending away from the normal as it enters the air from the water		1 point
For ray A bending the opposite direction in relationship to the normal as it exits the air and enters the water compared to the refraction entering the air from the water		1 point
Note: The normals need not be shown.		

(b) LO 5.B.4.1, SP 6.4, 7.2; LO 5.B.4.2, SP 1.4, 7.2; LO 5.B.5.4, SP 6.4, 2.2; LO 5.B.5.5, SP 2.2, 6.4
3 points

The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

For a valid application of the work-energy theorem		1 point
$\Delta K = W_{\text{net}} = \lvert W_b \rvert - \lvert W_g \rvert - \lvert W_{\text{diss}} \rvert$		
For finding the work done by the buoyant force		1 point
$W_b = \rho_w V_1 g h$		
For correct substitutions into an equation with consistent relative signs for the terms		1 point
$\frac{1}{2}\rho_A V_1 v_f^2 = \rho_W V_1 g h - \rho_A V_1 g h - \lvert W_{\text{diss}} \rvert$		
$\lvert W_{\text{diss}} \rvert = \rho_W V_1 g h - \rho_A V_1 g h - \frac{1}{2}\rho_A V_1 v_f^2$		

Question 4 (continued)

(c)

At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \mathrm{kg/m^3}$.

i. LO 5.B.10.1, SP 2.2
1 point

Determine the absolute pressure in the bubble at this location.

$P_{4.5\mathrm{m}} = P_{atm} + \rho_w g d$		
$P_{4.5\mathrm{m}} = \left(1.0 \times 10^5 \mathrm{Pa}\right) + \left(1000\ \mathrm{kg/m^3}\right)\left(9.8\ \mathrm{m/s^2}\right)(4.5\ \mathrm{m})$		
For a correct answer with units		1 point
$P_{4.5\mathrm{m}} = 1.44 \times 10^5\ \mathrm{Pa}$ (or $1.45 \times 10^5\ \mathrm{Pa}$ using $g = 10\ \mathrm{m/s^2}$)		

ii. LO 7.A.3.3, SP 5.1
2 points

The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1 , calculate the volume of the bubble when it is just below the surface of the water.

For applying the ideal gas law at two locations in an attempt to determine the new bubble volume		1 point
$P_{4.5\mathrm{m}} V_1 = P_{atm} V_{surface}$		
$V_{surface} = P_{4.5\mathrm{m}} V_1 / P_{atm}$		
For substituting pressures consistent with part (i)		1 point
$V_{surface} = \left(1.44 \times 10^5\ \mathrm{Pa}\right) V_1 / \left(1 \times 10^5\ \mathrm{Pa}\right)$		
$V_{surface} = 1.44 V_1$ (or $1.45 V_1$ using $g = 10\ \mathrm{m/s^2}$)		

iii. LO 7.A.3.3, SP 5.1
1 point

If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

For a correct explanation		1 point
Note: The explanation may be qualitative or quantitative. The explanation may also be macroscopic or microscopic.		

Question 4 (continued)

(c) (continued)
iii. (continued)

Example 1: By the ideal gas law, $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}$, so $V_{surface} = P_{4.5\text{m}}V_1T_{surface}/P_{atm}T_1$. The two pressures still have their previous values. $T_{surface} < T_1$, so the volume at the surface will be smaller.		
Example 2: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to expand less than it did in the constant temperature situation or contract.		
Claim (given): The volume of the bubble will decrease Example 1 evidence: $P_{4.5\text{m}}V_1/T_1 = P_{atm}V_{surface}/T_{surface}$, so $V_{surface} = P_{4.5\text{m}}V_1T_{surface}/P_{atm}T_1$ Example 1 reasoning: The two pressures still have their previous values. $T_{surface} < T_1$, so the volume at the surface will be smaller. Example 2 evidence: As the bubble cools, the air molecules move slower. Slower molecules exert less force on the inner surface of the bubble. Example 2 reasoning: The unbalanced force, due to the difference in the forces on the inside and outside of the bubble, causes the bubble to contract.		

Learning Objectives:

LO 5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. [See Science Practices 6.4, 7.2]

LO 5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system using information from representations of that system. [See Science Practices 1.4, 2.1, 2.2]

LO 5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [See Science Practices 6.4, 7.2]

LO 5.B.5.5: The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. [See Science Practices 2.2, 6.4]

LO 5.B.10.1: The student is able to make calculations related to a moving fluid using Bernoulli's equation. [See Science Practices 2.2]

LO 6.E.3.1: The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. [See Science Practices 1.1, 1.4]

LO 7.A.3.3: The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law $PV = nRT$. [See Science Practices 5.1]