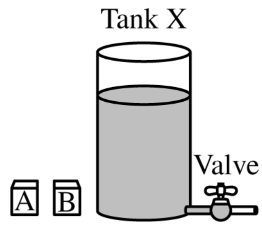
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

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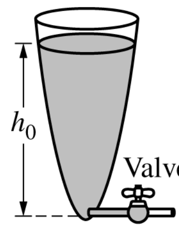
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

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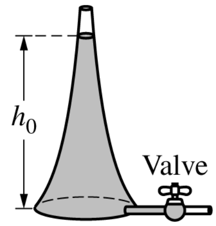
Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

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Question 3: Quantitative/Qualitative Translation — 12 points

(a)(i)	For a statement that collisions from the water particles exert upward forces on the block and collisions from the air particles exert downward forces on the block	**1 point**
	For a statement indicating that the force from the water is greater than the force from the air	**1 point**

Example Response

The air particles collide with the top of the block and exert downward forces on the block. The water particles collide with the bottom of the block and exert upward forces on the block. The force exerted by the water particles is greater than the force exerted by the air particles. Therefore, the result of these forces is an upward buoyant force from the particles.

(a)(ii)	For indicating that Block A has a greater density than Block B because Block A displaces a larger volume of water, thus the buoyant force on Block A is greater than the buoyant force on Block B	**1 point**

Example Response

Because Block A *displaces a greater volume of fluid, the buoyant force on Block* A *is greater than the buoyant force on Block* B. *Because the buoyant force and gravitational force are balanced for both blocks, Block* A *must weigh more than Block* B. *Because the blocks have the same volume, Block* A *is more dense than Block* B.

Total for part (a) — 3 points

(b)(i)	For using Bernoulli's equation to derive the relationship between v_p and h	**1 point**
	For indicating that $P_2 = P_1$	**1 point**
	For correct substitutions of the heights and speeds	**1 point**

Example Solution

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_s^2 + \rho g h = \rho g(0) + \frac{1}{2}\rho v_p^2$$

$$v_p^2 = v_s^2 + 2gh$$

$$v_p = \sqrt{v_s^2 + 2gh}$$

(b)(ii)	For using the continuity equation to derive the relationship between v_p and R	**1 point**
	For correct substitutions for the expressions of areas and speeds	**1 point**

Example Solution

$A_1 v_1 = A_2 v_2$

$\pi R^2 v_s = \pi r^2 v_p$

$v_p = \dfrac{R^2}{r^2} v_s$

(b)(iii)	For using conservation principles to justify that when $R \gg r$, then $v_s \ll v_p$	**1 point**
	For indicating a very small value of v_s will have a negligible effect on v_p	**1 point**

Example Response

When the cross sectional area of the tank is very large compared to the cross sectional area of the pipe, the speed v_s of the surface of the water is much less than the speed of the water v_p exiting the pipe due to the constant volume flow rate. As a result, the speed of the surface of the water can be approximated as zero, so the speed of the water exiting the pipe can be approximated as $v_p = \sqrt{2gh}$.

Total for part (b) 7 points

(c)	For correctly relating the decrease in v_p to the decrease in the height of the surface of the water h	**1 point**
	For correctly relating the decrease in v_s to the increase in radius R	**1 point**

Example Response

According to the equation in part (b)(i), $v_p = \sqrt{v_s^2 + 2gh}$. As h decreases, v_p decreases. When solving the equation in part (b)(ii) for v_s, it can be shown that $v_s = \dfrac{r^2}{R^2} v_p$. Therefore, an increase in R results in a decrease in v_s. Because v_p decreases with decreasing h, by using the same expression from part (b)(ii) in the case in which v_p decreases and R increases, it can be shown that the speed v_s of the water at the surface decreases.

OR

If the two equations from parts (b)(i) and (b)(ii) are solved simultaneously for v_s as a function of h and R, it can be shown that $v_s = \sqrt{\dfrac{2gh}{\dfrac{R^4}{r^4} - 1}}$. Therefore, as h decreases and R increases, v_s decreases.

Total for part (c) 2 points

Total for question 3 12 points