Begin your response to **QUESTION 3** on this page.

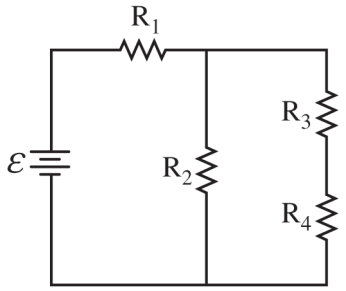

Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors R_1, R_2, R_3, and R_4, each of resistance R, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and R only.

i. **Derive** an expression for the current I_1 in Resistor R_1.

ii. **Derive** an expression for the current I_3 in Resistor R_3.

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(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.

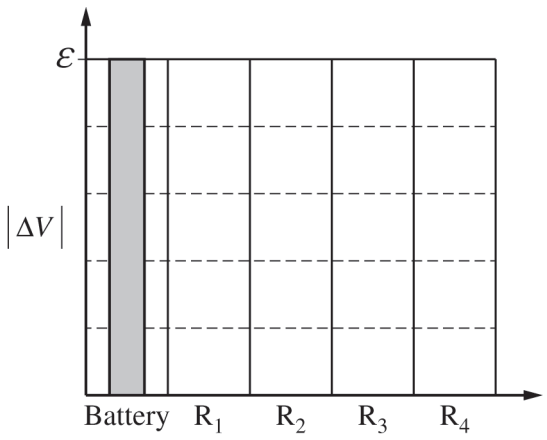

Figure 2

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \frac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for P is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

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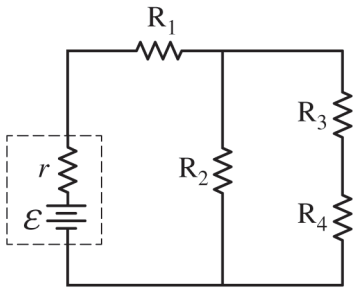

Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor R_1 is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance r to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor R_1 in the new circuit is P_{new}.

(d) **Indicate** whether P_{new} is greater than, less than, or equal to $P_{original}$.

_____ $P_{new} > P_{original}$ _____ $P_{new} < P_{original}$ _____ $P_{new} = P_{original}$

Briefly **justify** your answer.

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Question 3: Quantitative/Qualitative Translation — 12 points

(a)(i) For correctly determining the total resistance R_{total} of the circuit — **1 point**

Example Response

$$R_{\text{total}} = \frac{5R}{3}$$

For a multi-step derivation that includes correct substitutions of $\mathcal{E}$ and R_{total} into the equation that describes Ohm's law, consistent with the first point of part (a)(i) — **1 point**

Example Response

$$I_1 = \frac{3\mathcal{E}}{5R}$$

Example Solution

Determine the total resistance of the circuit.

The resistance of the right-most branch containing resistors connected in series:

$$R_s = \sum_i R_i$$
$$R_s = R + R$$
$$R_s = 2R$$

The resistance of parallel branches that contain resistors:

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$
$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$
$$R_p = \frac{2R}{3}$$

The total resistance of the circuit:

$$R_s = \sum_i R_i$$
$$R_{\text{total}} = R + \frac{2R}{3} = \frac{5R}{3}$$

The total current in the circuit:

$$I = \frac{\Delta V}{R} = \frac{3\mathcal{E}}{5R}$$

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(c) For indicating that the equation is correct or incorrect, consistent with the derivations in part (a) or the bar chart from part (b), with an attempt at a relevant justification **1 point**

For a justification that correctly relates P to at least **one** of the following: **1 point**

- The potential difference across the battery
- The current in the battery
- R_1, R_2, R_3, and R_4

For a justification that relates the equation $P = \frac{3\mathcal{E}^2}{5R}$ to at least **one** of the following: **1 point**

- The bar chart from the response in part (b)
- The derivations in the responses to parts (a)(i) and (a)(ii)

Scoring Note: The justification must be consistent with the derivations in part (a) or the bar chart from part (b).

Example Solution

The equation is correct. The equation for P, which is power, can be written as $P = \frac{(\Delta V)^2}{R}$. According to the bar chart in part (b), the potential difference across the battery is $\mathcal{E}$. The total resistance of the circuit is $\frac{5R}{3}$, according to the derivation from part (a)(i). Therefore, $P = \frac{(\Delta V)^2}{R} = \frac{(\mathcal{E})^2}{\left(\frac{5R}{3}\right)} = \frac{3\mathcal{E}^2}{5R}$.

Total for part (c) **3 points**

(d) For selecting that $P_{\text{new}} < P_{\text{original}}$ with an attempt at a relevant justification **1 point**

For a correct justification that indicates at least **one** of the following: **1 point**

- The current in R_1 is less in the new circuit than in the original circuit
- The potential difference across R_1 is less in the new circuit than in the original circuit

Example Response

$P_{\text{new}} < P_{\text{original}}$. Since the emf *of the battery is the same in the new circuit and the total resistance of the new circuit is greater, the current in R_1 is less in the new circuit. Therefore, P_{new} is less than P_{original}.*

Total for part (d) **2 points**

Total for question 3 **12 points**

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